STPM/S(E)950

MAJLIS PEPERIKSAAN MALAYSIA

(*MALAYSIAN EXAMINATIONS COUNCIL*)

PEPERIKSAAN
SIJIL TINGGI PERSEKOLAHAN MALAYSIA

(*MALAYSIA HIGHER SCHOOL CERTIFICATE EXAMINATION*)

MATHEMATICS (M)

Syllabus and Specimen Papers

This syllabus applies for the 2012/2013 session and thereafter until further notice.

NATIONAL EDUCATION PHILOSOPHY

"Education in Malaysia is an on-going effort towards further developing the potential of individuals in a holistic and integrated manner, so as to produce individuals who are intellectually, spiritually, emotionally and physically balanced and harmonious, based on a belief in and devotion to God. Such effort is designed to produce Malaysian citizens who are knowledgeable and competent, who possess high moral standards, and who are responsible and capable of achieving a high level of personal well-being as well as being able to contribute to the betterment of the family, the society and the nation at large."

FOREWORD

This revised Mathematics (M) syllabus is designed to replace the existing syllabus which has been in use since the 2002 STPM examination. This new syllabus will be enforced in 2012 and the first examination will also be held the same year. The revision of the syllabus takes into account the changes made by the Malaysian Examinations Council (MEC) to the existing STPM examination. Through the new system, the form six study will be divided into three terms, and candidates will sit for an examination at the end of each term. The new syllabus fulfils the requirements of this new system. The main objective of introducing the new examination system is to enhance the teaching and learning orientation of form six so as to be in line with the orientation of teaching and learning in colleges and universities.

The Mathematics (M) syllabus is designed to provide a framework for a pre-university course that enables candidates to develop the understanding of mathematical concepts and mathematical thinking, and acquire skills in problem solving and the applications of mathematics related to social sciences and management. The assessment tools of this syllabus consist of written papers and coursework. Coursework offers opportunities for candidates to conduct mathematical investigation and mathematical modelling that enhance their understanding of mathematical processes and applications and provide a platform for them to develop soft skills.

The syllabus contains topics, teaching periods, learning outcomes, examination format, performance descriptions and specimen papers.

The design of this syllabus was undertaken by a committee chaired by Professor Dr. Abu Osman bin Md Tap from International Islamic University Malaysia. Other committee members consist of university lecturers, representatives from the Curriculum Development Division, Ministry of Education Malaysia, and experienced teachers who are teaching Mathematics. On behalf of MEC, I would like to thank the committee for their commitment and invaluable contribution. It is hoped that this syllabus will be a guide for teachers and candidates in the teaching and learning process.

Chief Executive
Malaysian Examinations Council

CONTENTS

Syllabus 950 Mathematics (M)

SYLLABUS

950 MATHEMATICS (M)

[*May not be taken with 954 Mathematics* (*T*)]

Aims

The Mathematics (M) syllabus is designed to provide a framework for a pre-university course that enables candidates to develop the understanding of mathematical concepts and mathematical thinking, and acquire skills in problem solving and the applications of mathematics related to social sciences and management.

Objectives

The objectives of the syllabus are to enable candidates to:

(*a*) use mathematical concepts, terminology and notation;

(*b*) display and interpret mathematical information in tabular, diagrammatic and graphical forms;

(*c*) identify mathematical patterns and structures in a variety of situations;

(*d*) use appropriate mathematical models in different contexts;

(*e*) apply mathematical principles and techniques in solving problems;

(*f*) carry out calculations and approximations to an appropriate degree of accuracy;

(*g*) interpret the significance and reasonableness of results;

(*h*) present mathematical explanations, arguments and conclusions.

FIRST TERM: ALGEBRA AND CALCULUS

Topic	*Teaching Period*	*Learning Outcome*
1 Functions	**20**	Candidates should be able to:
1.1 Functions	6	(*a*) state the domain and range of a function, and find composite functions; (*b*) determine whether a function is one-to-one, and find the inverse of a one-to-one function; (*c*) sketch the graphs of simple functions, including piecewise-defined functions;
1.2 Polynomial and rational functions	8	(*d*) use the factor theorem and the remainder theorem; (*e*) solve polynomial and rational equations and inequalities; (*f*) solve equations and inequalities involving modulus signs in simple cases; (*g*) decompose a rational expression into partial fractions in cases where the denominator has two distinct linear factors, or a linear factor and a prime quadratic factor;
1.3 Exponential and logarithmic functions	6	(*h*) relate exponential and logarithmic functions, algebraically and graphically; (*i*) use the properties of exponents and logarithms; (*j*) solve equations and inequalities involving exponential or logarithmic expressions.
2 Sequences and Series	**18**	Candidates should be able to:
2.1 Sequences	4	(*a*) use an explicit formula and a recursive formula for a sequence; (*b*) find the limit of a convergent sequence;
2.2 Series	8	(*c*) use the formulae for the *n*th term and for the sum of the first *n* terms of an arithmetic series and of a geometric series; (*d*) identify the condition for the convergence of a geometric series, and use the formula for the sum of a convergent geometric series; (*e*) use the method of differences to find the *n*th partial sum of a series, and deduce the sum of the series in the case when it is convergent;

Topic	*Teaching Period*	*Learning Outcome*
2.3 Binomial expansions	6	(*f*) expand $(a + b)^n$, where $n \in \mathbb{Z}^+$; (*g*) expand $(1 + x)^n$, where $n \in \mathbb{Q}$, and identify the condition $\|x\| < 1$ for the validity of this expansion; (*h*) use binomial expansions in approximations.
3 Matrices	**16**	Candidates should be able to:
3.1 Matrices	10	(*a*) identify null, identity, diagonal, triangular and symmetric matrices; (*b*) use the conditions for the equality of two matrices; (*c*) perform scalar multiplication, addition, subtraction and multiplication of matrices with at most three rows and three columns; (*d*) use the properties of matrix operations; (*e*) find the inverse of a non-singular matrix using elementary row operations; (*f*) evaluate the determinant of a matrix; (*g*) use the properties of determinants;
3.2 Systems of linear equations	6	(*h*) reduce an augmented matrix to row-echelon form, and determine whether a system of linear equations has a unique solution, infinitely many solutions or no solution; (*i*) apply the Gaussian elimination to solve a system of linear equations; (*j*) find the unique solution of a system of linear equations using the inverse of a matrix.
4 Differentiation	**20**	Candidates should be able to:
4.1 Limits	6	(*a*) determine the existence and values of the left-hand limit, right-hand limit and limit of a function; (*b*) use the properties of limits;
4.2 Derivatives	6	(*c*) identify the derivative of a function as a limit; (*d*) find the derivatives of x^n ($n \in \mathbb{Q}$), e^x, $\ln x$, with constant multiples, sums, differences, products, quotients and composites; (*e*) perform implicit differentiation.

Topic	*Teaching Period*	*Learning Outcome*
4.3 Applications of differentiation	8	(*f*) determine where a function is increasing, decreasing, concave upward and concave downward; (*g*) determine the stationary points, extremum points and points of inflexion; (*h*) sketch the graphs of functions, including asymptotes parallel to the coordinate axes.
5 Integration	**20**	Candidates should be able to:
5.1 Indefinite integrals	10	(*a*) identify integration as the reverse of differentiation; (*b*) integrate x^n ($n \in \mathbb{Q}$) and e^x, with constant multiples, sums and differences; (*c*) integrate rational functions by means of decomposition into partial fractions; (*d*) use algebraic substitutions to find integrals; (*e*) perform integration by parts;
5.2 Definite integrals	10	(*f*) identify a definite integral as the area under a curve; (*g*) use the properties of definite integrals; (*h*) evaluate definite integrals; (*i*) calculate the area of a region bounded by a curve and lines parallel to the coordinate axes, or between two curves; (*j*) calculate volumes of solids of revolution about one of the coordinate axes.
6 Differential Equations	**14**	Candidates should be able to: (*a*) find the general solution of a first order differential equation with separable variables; (*b*) find the general solution of a first order linear differential equation by means of an integrating factor; (*c*) transform, by a given substitution, a first order differential equation into one with separable variables or one which is linear; (*d*) use a boundary condition to find a particular solution; (*e*) solve problems, related to social sciences and management, that can be modelled by differential equations.

SECOND TERM: STATISTICS

Topic	*Teaching Period*	*Learning Outcome*
7 Data Description	**14**	Candidates should be able to: (*a*) identify discrete, continuous, ungrouped and grouped data; (*b*) construct and interpret stem-and-leaf diagrams, box-and-whisker plots, histograms and cumulative frequency curves; (*c*) state the mode and range of ungrouped data; (*d*) determine the median and interquartile range of ungrouped and grouped data; (*e*) calculate the mean and standard deviation of ungrouped and grouped data from raw data and from given totals such as $\sum_{i=1}^{n}(x_i - a)$ and $\sum_{i=1}^{n}(x_i - a)^2$; (*f*) select and use the appropriate measures of central tendency and measures of dispersion; (*g*) calculate the Pearson coefficient of skewness; (*h*) describe the shape of a data distribution.
8 Probability	**14**	Candidates should be able to: (*a*) apply the addition principle and the multiplication principle; (*b*) use the formulae for combinations and permutations in simple cases; (*c*) identify a sample space, and calculate the probability of an event; (*d*) identify complementary, exhaustive and mutually exclusive events; (*e*) use the formula $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$; (*f*) calculate conditional probabilities, and identify independent events; (*g*) use the formulae $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B \mid A) = \mathrm{P}(B) \times \mathrm{P}(A \mid B)$; (*h*) use the rule of total probability.

Topic	*Teaching Period*	*Learning Outcome*
9 Probability Distributions	**26**	Candidates should be able to:
9.1 Discrete random variables	6	(*a*) identify discrete random variables; (*b*) construct a probability distribution table for a discrete random variable; (*c*) use the probability function and cumulative distribution function of a discrete random variable; (*d*) calculate the mean and variance of a discrete random variable;
9.2 Continuous random variables	6	(*e*) identify continuous random variables; (*f*) relate the probability density function and cumulative distribution function of a continuous random variable; (*g*) use the probability density function and cumulative distribution function of a continuous random variable; (*h*) calculate the mean and variance of a continuous random variable;
9.3 Binomial distribution	4	(*i*) use the probability function of a binomial distribution, and find its mean and variance; (*j*) use the binomial distribution as a model for solving problems related to social sciences and management;
9.4 Poisson distribution	4	(*k*) use the probability function of a Poisson distribution, and identify its mean and variance; (*l*) use the Poisson distribution as a model for solving problems related to social sciences and management;
9.5 Normal distribution	6	(*m*) identify the general features of a normal distribution, in relation to its mean and standard deviation; (*n*) standardise a normal random variable, and use the normal distribution tables; (*o*) use the normal distribution as a model for solving problems related to social sciences and management; (*p*) use the normal distribution, with continuity correction, as an approximation to the binomial distribution, where appropriate.

Topic	*Teaching Period*	*Learning Outcome*
10 Correlation and Regression	**22**	Candidates should be able to:
10.1 Correlation	8	(*a*) plot a scatter diagram, and describe the correlation between two variables; (*b*) identify a spurious correlation; (*c*) calculate and interpret the Pearson correlation coefficient; (*d*) calculate and interpret the Spearman rank correlation coefficient;
10.2 Regression	14	(*e*) find the equation of a least squares regression line; (*f*) interpret regression coefficients; (*g*) use a regression line to make predictions; (*h*) calculate and interpret the coefficient of determination; (*i*) use transformation of variables to obtain linearity in regression analysis.
11 Index Numbers	**12**	Candidates should be able to: (*a*) explain the meaning of an index number; (*b*) calculate and interpret the simple average of relatives; (*c*) calculate and interpret the weighted average of relatives; (*d*) calculate and interpret simple aggregate indices; (*e*) calculate and interpret Laspeyres and Paasche indices.
12 Time Series	**20**	Candidates should be able to: (*a*) identify the components of a time series; (*b*) identify an additive and a multiplicative time series model; (*c*) plot and comment on a time series; (*d*) use the method of moving averages to smooth a time series; (*e*) calculate and interpret seasonal indices; (*f*) obtain a seasonally adjusted time series; (*g*) use the least squares method to fit a trend line; (*h*) use a trend line for forecasting.

THIRD TERM: FINANCIAL AND DECISION MATHEMATICS

Topic	*Teaching Period*	*Learning Outcome*
13 Interest, Annuity and Depreciation	**20**	Candidates should be able to:
13.1 Interest	8	(*a*) calculate simple interest and compound interest; (*b*) calculate the compound amount involving a number of interest periods; (*c*) determine an effective interest rate; (*d*) solve problems related to savings and loans;
13.2 Annuity	8	(*e*) calculate the future value of an annuity; (*f*) solve problems related to sinking funds; (*g*) calculate the present value of an annuity; (*h*) solve problems related to amortisation;
13.3 Depreciation	4	(*i*) use the flat rate depreciation method; (*j*) use the reducing balance depreciation method.
14 Cost, Revenue and Profit	**16**	Candidates should be able to: (*a*) find cost, revenue and profit functions, and use their relationship; (*b*) determine the break-even point; (*c*) find average cost, average revenue and average profit functions; (*d*) find marginal cost, marginal revenue and marginal profit functions; (*e*) determine the minimum cost, maximum revenue and maximum profit; (*f*) find demand and supply functions; (*g*) determine the equilibrium price and equilibrium demand; (*h*) calculate and interpret the consumer's surplus and producer's surplus.

Topic	*Teaching Period*	*Learning Outcome*
15 Linear Programming	**20**	Candidates should be able to:
15.1 Problem formulation	4	(*a*) determine the constraints; (*b*) find an objective function; (*c*) state a linear programming model;
15.2 Graphical method	8	(*d*) identify a feasible region; (*e*) determine the optimal solution; (*f*) identify special cases: infeasibility, unbounded solutions and multiple optimal solutions;
15.3 Simplex method	8	(*g*) obtain the standard simplex form using slack and/or surplus variables; (*h*) construct simplex tableaux for a maximisation problem with at most three variables and three constraints in addition to non-negativity conditions; (*i*) determine the optimal solution.
16 Critical Path Analysis	**20**	Candidates should be able to:
16.1 Networks	4	(*a*) identify activities on arcs and activities on nodes; (*b*) construct an activity network for a project;
16.2 Critical paths	8	(*c*) calculate the earliest and latest start and finish times for activities; (*d*) calculate and interpret the total float for an activity; (*e*) identify critical activities and critical paths; (*f*) determine the minimum completion time of a project;
16.3 Scheduling and crashing	8	(*g*) construct a Gantt chart and resource histogram for a project; (*h*) determine the minimum number of workers to complete a project in a given time; (*i*) determine the minimum time to complete a project for a given number of workers;

Topic	*Teaching Period*	*Learning Outcome*
		(*j*) determine the effect of adjusting the duration of an activity on the critical path and completion time; (*k*) determine the effect of adjusting the number of workers on the critical path and completion time.
17 Inventory Models	**16**	Candidates should be able to: (*a*) identify assumptions of the basic economic order quantity model; (*b*) use the basic economic order quantity model; (*c*) use the economic order quantity model with planned shortages; (*d*) find the holding cost and ordering cost; (*e*) find the reorder point, cycle time and number of orders per year; (*f*) determine the minimum total cost order quantity; (*g*) determine the minimum total cost per year.
18 Game Theory	**16**	Candidates should be able to: (*a*) explain the meaning of a two-person zero-sum game; (*b*) determine the outcomes for players from a given pay-off matrix; (*c*) determine play-safe strategies and identify a stable solution; (*d*) find the expected value of a game; (*e*) use a dominance argument to reduce a pay-off matrix; (*f*) determine optimal mixed strategies for a game with no stable solution; (*g*) use a graphical method to solve a game, where appropriate.

Coursework

The Mathematics (M) coursework is intended to enable candidates to carry out mathematical investigation and mathematical modelling, so as to enhance the understanding of mathematical processes and applications and to develop soft skills.

The coursework comprises three assignments set down by the Malaysian Examinations Council. The assignments are based on three different areas of the syllabus and represent two types of tasks: mathematical investigation and mathematical modelling.

A school candidate is required to carry out one assignment in each term under the supervision of the subject teacher as specified in the Teacher's Manual for Mathematics (M) Coursework which can be downloaded from MEC's Portal (http://www.mpm.edu.my) by the subject teacher during the first term of form six. The assignment reports are graded by the subject teacher in the respective terms. A viva session is conducted by the teacher in each term after the assessment of the assignment reports.

An individual private candidate is required to carry out one assignment in each term as specified in the Individual Private Candidate's Manual for Mathematics (M) Coursework which can be downloaded from MEC's Portal (http://www.mpm.edu.my) by the candidate during the first term of form six. The assignment reports are graded by an external examiner in the respective terms. A viva session is conducted by the examiner in each term after the assessment of the assignment reports.

A repeating candidate may use the total mark obtained in the coursework for the subsequent STPM examination. Requests to carry forward the moderated coursework mark should be made during the registration of the examination.

Scheme of Assessment

Term of Study	*Paper Code and Name*	*Type of Test*	*Mark* (*Weighting*)	*Duration*	*Administration*
First Term	950/1 Mathematics (M) Paper 1	**Written test** **Section A** Answer all 6 questions of variable marks. **Section B** Answer 1 out of 2 questions. All questions are based on topics 1 to 6.	**60 (26.67%)** 45 15	1½ hours	Central assessment
Second Term	950/2 Mathematics (M) Paper 2	**Written test** **Section A** Answer all 6 questions of variable marks. **Section B** Answer 1 out of 2 questions. All questions are based on topics 7 to 12.	**60 (26.67%)** 45 15	1½ hours	Central assessment
Third Term	950/3 Mathematics (M) Paper 3	**Written test** **Section A** Answer all 6 questions of variable marks. **Section B** Answer 1 out of 2 questions. All questions are based on topics 13 to 18.	**60 (26.67%)** 45 15	1½ hours	Central assessment
First, Second and Third Terms	950/4 Mathematics (M) Paper 4	**Coursework** 3 assignments, each based on topics 1 to 6, topics 7 to 12 and topics 13 to 18.	**180** to be scaled to **45 (20%)**	Throughout the three terms	Assessment by school teachers for candidates from government and government-aided schools Assessment by appointed assessors for candidates from private schools and individual private candidates

Performance Descriptions

A grade **A** candidate is likely able to:

(*a*) use correctly mathematical concepts, terminology and notation;

(*b*) display and interpret mathematical information in tabular, diagrammatic and graphical forms;

(*c*) identify mathematical patterns and structures in a variety of situations;

(*d*) use appropriate mathematical models in different contexts;

(*e*) apply correctly mathematical principles and techniques in solving problems;

(*f*) carry out calculations and approximations to an appropriate degree of accuracy;

(*g*) interpret the significance and reasonableness of results, making sensible predictions where appropriate;

(*h*) present mathematical explanations, arguments and conclusions, usually in a logical and systematic manner.

A grade **C** candidate is likely able to:

(*a*) use correctly some mathematical concepts, terminology and notation;

(*b*) display and interpret some mathematical information in tabular, diagrammatic and graphical forms;

(*c*) identify mathematical patterns and structures in certain situations;

(*d*) use appropriate mathematical models in certain contexts;

(*e*) apply correctly some mathematical principles and techniques in solving problems;

(*f*) carry out some calculations and approximations to an appropriate degree of accuracy;

(*g*) interpret the significance and reasonableness of some results;

(*h*) present some mathematical explanations, arguments and conclusions.

Mathematical Notation

Miscellaneous symbols

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$<$	is less than
$\leqslant$	is less than or equal to
$>$	is greater than
$\geqslant$	is greater than or equal to
∞	infinity
$\therefore$	therefore

Operations

$a + b$	a plus b
$a - b$	a minus b
$a \times b$, ab	a multiplied by b
$a \div b$, $\frac{a}{b}$	a divided by b
$a : b$	ratio of a to b
a^n	nth power of a
$a^{\frac{1}{2}}$, $\sqrt{a}$	positive square root of a
$a^{\frac{1}{n}}$, $\sqrt[n]{a}$	positive nth root of a
$\lvert a \rvert$	absolute value of a real number a
$\sum_{i=1}^{n} u_i$	$u_1 + u_2 + \cdots + u_n$
$n!$	n factorial for $n \in \mathbb{N}$
$\binom{n}{r}$	binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n, r \in \mathbb{N}$, $0 \leqslant r \leqslant n$

Set notation

$\in$	is an element of
$\notin$	is not an element of
$\varnothing$	empty set
$\{ x \mid \ldots \}$	set of x such that . . .
$\mathbb{N}$	set of natural numbers, $\{0, 1, 2, 3, \ldots\}$
$\mathbb{Z}$	set of integers
$\mathbb{Z}^+$	set of positive integers
$\mathbb{Q}$	set of rational numbers
$\mathbb{R}$	set of real numbers

$[\,a, b\,]$	closed interval $\{x \mid x \in \mathbb{R}\,,\, a \leqslant x \leqslant b\}$
$(\,a, b\,)$	open interval $\{x \mid x \in \mathbb{R}\,,\, a < x < b\}$
$[\,a, b\,)$	interval $\{x \mid x \in \mathbb{R}\,,\, a \leqslant x < b\}$
$(\,a, b\,]$	interval $\{x \mid x \in \mathbb{R}\,,\, a < x \leqslant b\}$
$\cup$	union
$\cap$	intersection

Functions

f	a function f
f(x)	value of a function f at x
$\mathrm{f} : A \to B$	f is a function under which each element of set A has an image in set B
$\mathrm{f} : x \mapsto y$	f is a function which maps the element x to the element y
f^{-1}	inverse function of f
$\mathrm{f} \circ \mathrm{g}$	composite function of f and g which is defined by $\mathrm{f} \circ \mathrm{g}(x) = \mathrm{f}[\mathrm{g}(x)]$
e^x	exponential function of x
$\log_a x$	logarithm to base a of x
$\ln x$	natural logarithm of x, $\log_\mathrm{e} x$

Matrices

$\mathbf{A}$	a matrix $\mathbf{A}$
$\mathbf{0}$	null matrix
$\mathbf{I}$	identity matrix
$\mathbf{A}^\mathrm{T}$	transpose of a matrix $\mathbf{A}$
$\mathbf{A}^{-1}$	inverse of a non-singular square matrix $\mathbf{A}$
det $\mathbf{A}$	determinant of a square matrix $\mathbf{A}$

Derivatives and integrals

$\lim_{x \to a} \mathrm{f}(x)$	limit of f(x) as x tends to a
$\dfrac{\mathrm{d}y}{\mathrm{d}x}$	first derivative of y with respect to x
$\mathrm{f}'(x)$	first derivative of f(x) with respect to x
$\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}$	second derivative of y with respect to x
$\mathrm{f}''(x)$	second derivative of f(x) with respect to x
$\dfrac{\mathrm{d}^n y}{\mathrm{d}x^n}$	nth derivative of y with respect to x
$\mathrm{f}^{(n)}(x)$	nth derivative of f(x) with respect to x
$\int y \, \mathrm{d}x$	indefinite integral of y with respect to x
$\int_a^b y \, \mathrm{d}x$	definite integral of y with respect to x for values of x between a and b

Data description

$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
$\overline{x}$	sample mean
s^2	sample variance, $s^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2$
μ	population mean
σ^2	population variance

Probability

A	an event A
A'	complement of an event A or the event not A
P(A)	probability of an event A
P($A \mid B$)	probability of event A given event B

Probability distributions

X	a random variable X
x	value of a random variable X
Z	standardised normal random variable
z	value of the standardised normal random variable Z
f(x)	value of the probability density function of a continuous random variable X
F(x)	value of the cumulative distribution function of a continuous random variable X
E(X)	expectation of a random variable X
Var(X)	variance of a random variable X
B(n, p)	binomial distribution with parameters n and p
Po(λ)	Poisson distribution with parameter λ
N(μ, σ^2)	normal distribution with mean μ and variance σ^2

Correlation and regression

r	correlation coefficient for a sample
r^2	coefficient of determination
S_{xx} , S_{yy}	sums of squared deviations from the means
S_{xy}	sum of the cross products of deviations from the means

Interest and annuity

P	principal
I	interest
A	accrued amount
r	nominal interest rate
r_{eff}	effective interest rate
t	time in years
n	number of compounding periods

i	interest rate per compounding period
A	future value of an annuity
R	payment at the end of each period
P	present value of an annuity

Inventory models

D	demand rate
C_h	holding cost per unit per time period
C_b	backorder cost per unit per time period
C_o	ordering cost per order
Q	order quantity
Q*	optimal order quantity

Electronic Calculators

During the written paper examination, candidates are advised to have standard scientific calculators which must be silent. Programmable and graphic display calculators are prohibited.

Reference Books

Teachers and candidates may use books specially written for the STPM examination and other reference books such as those listed below.

Algebra and Calculus

1. Harcet, J., Heinrichs, L., Seiler, P.M. and Skoumal, M.T., 2012. *Mathematics: Higher Level, IB Diploma Programme*. United Kingdom: Oxford University Press.
2. Neill, H. and Quadling, D., 2002. *Advanced Level Mathematics: Pure Mathematics 1 and 2 & 3.* United Kingdom: Cambridge University Press.
3. Beecher, J.A., Penna, J.A. and Bittinger, M.L., 2012. *College Algebra*. 4th edition. Singapore: Pearson Addison-Wesley.
4. Blitzer, R., 2010. *College Algebra: An Early Functions Approach*. 2nd edition. Singapore: Pearson Prentice Hall.
5. Stewart, J., 2012. *Single Variable Calculus: Early Transcendentals.* 7th edition, Metric Version. Singapore: Brooks/Cole, Cengage Learning.
6. Tan, S.T., 2011. *Single Variable Calculus: Early Transcendentals*. Singapore: Brooks/Cole, Cengage Learning.

Statistics

7. Crawshaw, J. and Chambers, J., 2001. *A Concise Course in Advanced Level Statistics*. 4th edition. United Kingdom: Nelson Thornes.
8. Upton, G. and Cook, I., 2001. *Introducing Statistics*. 2nd edition. United Kingdom: Oxford University Press.
9. Curwin, J. and Slater, R., 2008. *Quantitative Methods for Business Decisions*. 6th edition. Singapore: South-Western, Cengage Learning.
10. Francis, A., 2008. *Business Mathematics and Statistics*. 6th edition. Singapore: South-Western, Cengage Learning.

Financial and Decision Mathematics

11. Lial, M.L., Greenwell, R.N. and Ritchey, N.P., 2012. *Finite Mathematics and Calculus with Applications*. 9th edition. Singapore: Pearson Addison-Wesley.
12. Sullivan, M. and Mizrahi A., 2004. *Mathematics: An Applied Approach*. 8th edition. Singapore: John Wiley.
13. Bloomfield I. and Stevens J., 2002. *Complete Advanced Level Mathematics: Discrete and Decision*. United Kingdom: Nelson Thornes.
14. Anderson, D.R. *et al*., 2011. *An Introduction to Management Science: Quantitative Approaches to Decision Making*. 13th edition. Singapore: South-Western, Cengage Learning.

SPECIMEN PAPER

950/1 STPM

MATHEMATICS (M) (MATEMATIK (M))

PAPER 1 (KERTAS 1)

One and a half hours (Satu jam setengah)

MAJLIS PEPERIKSAAN MALAYSIA
(MALAYSIAN EXAMINATIONS COUNCIL)

SIJIL TINGGI PERSEKOLAHAN MALAYSIA
(MALAYSIA HIGHER SCHOOL CERTIFICATE)

Instruction to candidates:

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Answer **all** *questions in Section A and any* **one** *question in Section B. Answers may be written in either English or Bahasa Malaysia.*

All necessary working should be shown clearly.

Scientific calculators may be used. Programmable and graphic display calculators are prohibited.

A list of mathematical formulae is provided on page of this question paper.

Arahan kepada calon:

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU UNTUK BERBUAT DEMIKIAN.

Jawab **semua** *soalan dalam Bahagian A dan mana-mana* **satu** *soalan dalam Bahagian B. Jawapan boleh ditulis dalam bahasa Inggeris atau Bahasa Malaysia.*

Semua kerja yang perlu hendaklah ditunjukkan dengan jelas.

Kalkulator sainstifik boleh digunakan. Kalkulator boleh atur cara dan kalkulator paparan grafik tidak dibenarkan.

Senarai rumus matematik dibekalkan pada halaman dalam kertas soalan ini.

This question paper consists of printed pages and blank page.
(Kertas soalan ini terdiri daripada halaman bercetak dan halaman kosong.)

Section A [45 *marks*]

Answer **all** *questions in this section.*

1 The function f is defined by $f(x) = \ln(1 - 2x)$, $x < 0$.

(*a*) Find f^{-1}, and state its domain. [3 *marks*]

(*b*) Sketch, on the same axes, the graphs of f and f^{-1}. [4 *marks*]

(*c*) Determine whether there is any value of x for which $f(x) = f^{-1}(x)$. [3 *marks*]

2 The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_{n+1} = 3u_n$, $u_1 = 2$.

(*a*) Write down the first five terms of the sequence. [2 *marks*]

(*b*) Suggest an explicit formula for u_r. [2 *marks*]

3 Using an augmented matrix and elementary row operations, find the solution of the system of equations

$$\begin{aligned} 3x - 2y - 5z &= -5 \\ x + 3y - 2z &= -6 \\ 5x - 4y + z &= 11 \end{aligned}$$ [9 *marks*]

4 Find the gradients of the curve $y^3 + y = x^3 + x^2$ at the points where the curve meets the coordinate axes. [6 *marks*]

5 Show that

$$\int_1^{\mathrm{e}} x \ln x \, \mathrm{d}x = \tfrac{1}{4}(\mathrm{e}^2 + 1).$$ [4 *marks*]

Hence, find the value of $\int_1^{\mathrm{e}} x(\ln x)^2 \, \mathrm{d}x$. [3 *marks*]

6 The variables x and y, where $x, y > 0$, are related by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + y^2 = -\frac{2y}{x}.$$

Using the substitution $y = \dfrac{u}{x^2}$, show that the differential equation may be reduced to

$$\frac{\mathrm{d}u}{\mathrm{d}x} = -\frac{u^2}{x^2}.$$ [3 *marks*]

Solve this differential equation, and hence, find y, in terms of x, with the condition that $y = 1$ when $x = 1$. [6 *marks*]

Bahagian A [45 *markah*]

Jawab **semua** *soalan dalam bahagian ini*.

1 Fungsi f ditakrifkan oleh $f(x) = \ln (1 - 2x)$, $x < 0$.

(*a*) Cari f^{-1}, dan nyatakan domainnya. [3 *markah*]

(*b*) Lakar, pada paksi yang sama, graf f dan f^{-1}. [4 *markah*]

(*c*) Tentukan sama ada terdapat sebarang nilai x untuk $f(x) = f^{-1}(x)$. [3 *markah*]

2 Jujukan $u_1, u_2, u_3, \ldots$ ditakrifkan oleh $u_{n+1} = 3u_n$, $u_1 = 2$.

(*a*) Tulis lima sebutan yang pertama jujukan itu. [2 *markah*]

(*b*) Cadangkan satu rumus tak tersirat bagi u_r. [2 *markah*]

3 Dengan menggunakan satu matriks imbuhan dan operasi baris permulaan, cari selesaian sistem persamaan

$$\begin{aligned} 3x - 2y - 5z &= -5 \\ x + 3y - 2z &= -6 \\ 5x - 4y + z &= 11 \end{aligned}$$ [9 *markah*]

4 Cari kecerunan lengkung $y^3 + y = x^3 + x^2$ di titik di mana lengkung itu bertemu dengan paksi koordinat. [6 *markah*]

5 Tunjukkan bahawa

$$\int_1^{\mathrm{e}} x \ln x \, \mathrm{d}x = \tfrac{1}{4}(\mathrm{e}^2 + 1).$$ [4 *markah*]

Dengan yang demikian, cari nilai $\int_1^{\mathrm{e}} x(\ln x)^2 \, \mathrm{d}x$. [3 *markah*]

6 Pembolahubah x dan y, dengan x, $y > 0$, dihubungkan oleh persamaan pembezaan

$$\frac{\mathrm{d}y}{\mathrm{d}x} + y^2 = -\frac{2y}{x}.$$

Dengan menggunakan gantian $y = \dfrac{u}{x^2}$, tunjukkan bahawa persamaan pembezaan itu boleh diturunkan kepada

$$\frac{\mathrm{d}u}{\mathrm{d}x} = -\frac{u^2}{x^2}.$$ [3 *markah*]

Selesaikan persamaan pembezaan ini, dan dengan yang demikian, cari y, dalam sebutan x, dengan syarat $y = 1$ apabila $x = 1$. [6 *markah*]

Section B [15 *marks*]

Answer any **one** *question in this section.*

7 Expand $(1+x)^{\frac{2}{3}}$ and $\frac{1+ax}{1+bx}$, where $|\ b\ | < 1$, in ascending powers of x up to the term in x^3. Determine the set of values of x for which both the expansions are valid. [7 *marks*]

If the two expansions are identical up to the term in x^2,

(*a*) determine the values of a and b, [3 *marks*]

(*b*) use $x = \frac{1}{8}$ to obtain the approximation $\sqrt[3]{81} \approx \frac{212}{49}$, [3 *marks*]

(*c*) find, correct to five decimal places, the difference between the terms in x^3 for the two expansions with $x = \frac{1}{8}$. [2 *marks*]

8 Sketch, on the same axes, the curve $y^2 = x$ and the straight line $y = 2 - x$, showing the coordinates of the points of intersection. [4 *marks*]

(*a*) State whether the curve $y^2 = x$ has a turning point. Justify your answer. [2 *marks*]

(*b*) Calculate the area of the region bounded by the curve $y^2 = x$ and the straight line $y = 2 - x$. [4 *marks*]

(*c*) Calculate the volume of the solid formed by revolving the region bounded by the curve $y^2 = x$ and the straight line $y = 2 - x$ completely about the y-axis. [5 *marks*]

Bahagian B [15 *markah*]

Jawab mana-mana **satu** *soalan dalam bahagian ini.*

7 Kembangkan $(1+x)^{\frac{2}{3}}$ dan $\frac{1+ax}{1+bx}$, dengan $|b| < 1$, dalam kuasa x menaik hingga sebutan dalam x^3. Tentukan set nilai x supaya kedua-dua kembangan itu sah. [7 *markah*]

Jika dua kembangan itu secaman hingga sebutan dalam x^2,

(*a*) tentukan nilai a dan b, [3 *markah*]

(*b*) gunakan $x = \frac{1}{8}$ untuk memperoleh penghampiran $\sqrt[3]{81} \approx \frac{212}{49}$, [3 *markah*]

(*c*) cari, betul hingga lima tempat perpuluhan, beza antara sebutan dalam x^3 bagi dua kembangan itu dengan $x = \frac{1}{8}$. [2 *markah*]

8 Lakar, pada paksi yang sama, lengkung $y^2 = x$ dan garis lurus $y = 2 - x$, dengan menunjukkan koordinat titik persilangan. [4 *markah*]

(*a*) Nyatakan sama ada lengkung $y^2 = x$ mempunyai titik pusingan. Justifikasikan jawapan anda. [2 *markah*]

(*b*) Hitung luas rantau yang dibatasi oleh lengkung $y^2 = x$ dan garis lurus $y = 2 - x$. [4 *markah*]

(*c*) Hitung isipadu pepejal yang terbentuk dengan mengisar rantau yang dibatasi oleh lengkung $y^2 = x$ dan garis lurus $y = 2 - x$ selengkapnya di sekitar paksi-y. [5 *markah*]

MATHEMATICAL FORMULAE (RUMUS MATEMATIK)

Binomial expansions (*Kembangan binomial*)

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n, \quad n \in \mathbb{Z}^+$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \cdots, \quad n \in \mathbb{Q}, |x| < 1$$

Integration (*Pengamiran*)

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$$

SPECIMEN PAPER

950/2 STPM

MATHEMATICS (M) (MATEMATIK (M))

PAPER 2 (KERTAS 2)

One and a half hours (Satu jam setengah)

MAJLIS PEPERIKSAAN MALAYSIA
(MALAYSIAN EXAMINATIONS COUNCIL)

SIJIL TINGGI PERSEKOLAHAN MALAYSIA
(MALAYSIA HIGHER SCHOOL CERTIFICATE)

Instruction to candidates:

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Answer **all** *questions in Section A and any* **one** *question in Section B. Answers may be written in either English or Bahasa Malaysia.*

All necessary working should be shown clearly.

Scientific calculators may be used. Programmable and graphic display calculators are prohibited.

A list of mathematical formulae, statistical tables and graph papers are provided on pages of this question paper.

Arahan kepada calon:

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU UNTUK BERBUAT DEMIKIAN.

Jawab **semua** *soalan dalam Bahagian A dan mana-mana* **satu** *soalan dalam Bahagian B. Jawapan boleh ditulis dalam bahasa Inggeris atau Bahasa Malaysia.*

Semua kerja yang perlu hendaklah ditunjukkan dengan jelas.

Kalkulator sainstifik boleh digunakan. Kalkulator boleh atur cara dan kalkulator paparan grafik tidak dibenarkan.

Senarai rumus matematik, jadual statistik dan kertas graf dibekalkan pada halaman dalam kertas soalan ini.

This question paper consists of printed pages and blank page.
(Kertas soalan ini terdiri daripada halaman bercetak dan halaman kosong.)

Section A [45 *marks*]

Answer **all** *questions in this section.*

1 The lengths (in seconds) of 60 songs recorded by a certain group of singers are summarised in the table below.

Song length (x)	*Number of songs*
$0 < x \leqslant 120$	1
$120 < x \leqslant 180$	9
$180 < x \leqslant 240$	15
$240 < x \leqslant 300$	17
$300 < x \leqslant 360$	13
$360 < x \leqslant 600$	5

(*a*) Display the data on a histogram. [3 *marks*]

(*b*) Calculate the mean song length to the nearest half second. [2 *marks*]

(*c*) Calculate the standard deviation of the song lengths to the nearest half second. [3 *marks*]

2 Two companies X and Y operate call-taxi services in a town. The percentages of residents in the town using the taxi services from companies X and Y are 40% and 60% respectively. The probabilities of taxis from companies X and Y being late are 0.02 and 0.01 respectively. A taxi is booked at random. Find the probability that

(*a*) the taxi is late and it is from company X, [2 *marks*]

(*b*) the taxi is late. [3 *marks*]

3 The probability density function of a continuous random variable X is given by

$$\mathrm{f}(x) = \begin{cases} ax(b-x), & 0 \leqslant x \leqslant b, \\ 0, & \text{otherwise}, \end{cases}$$

where a and b are positive constants. It has a value of $\frac{1}{2}$ at the mean value of X.

(*a*) Determine the values of a and b. [7 *marks*]

(*b*) Sketch the graph of the probability density function. [2 *marks*]

Bahagian A [45 *markah*]

Jawab **semua** *soalan dalam bahagian ini*.

1 Panjang (dalam saat) 60 lagu yang dirakamkan oleh sekumpulan penyanyi diiktisarkan dalam jadual di bawah.

Panjang lagu (x)	*Bilangan lagu*
$0 < x \leqslant 120$	1
$120 < x \leqslant 180$	9
$180 < x \leqslant 240$	15
$240 < x \leqslant 300$	17
$300 < x \leqslant 360$	13
$360 < x \leqslant 600$	5

(*a*) Paparkan data itu pada satu histogram. [3 *markah*]

(*b*) Hitung min panjang lagu hingga setengah saat terdekat. [2 *markah*]

(*c*) Hitung sisihan piawai panjang lagu itu hingga setengah saat terdekat. [3 *markah*]

2 Dua syarikat *X* dan *Y* mengusahakan perkhidmatan teksi panggilan di sebuah bandar. Peratusan penduduk di bandar itu yang menggunakan perkhidmatan teksi dari syarikat *X* dan *Y* masing-masing ialah 40% dan 60%. Kebarangkalian teksi dari syarikat *X* dan *Y* lewat masing-masing ialah 0.02 dan 0.01. Satu teksi ditempah secara rawak. Cari kebarangkalian bahawa

(*a*) teksi itu lewat dan teksi itu adalah dari syarikat *X*, [2 *markah*]

(*b*) teksi itu lewat. [3 *markah*]

3 Fungsi ketumpatan kebarangkalian pembolehubah rawak selanjar *X* diberikan oleh

$$\mathrm{f}(x) = \begin{cases} ax(b-x), & 0 \leqslant x \leqslant b, \\ 0, & \text{jika tidak,} \end{cases}$$

dengan *a* dan *b* pemalar positif. Fungsi itu mempunyai nilai $\frac{1}{2}$ pada min nilai *X*.

(*a*) Tentukan nilai *a* dan *b*. [7 *markah*]

(*b*) Lakar graf fungsi ketumpatan kebarangkalian itu. [2 *markah*]

4 A study is conducted to assess the impact of the size of a stall, x (in m^2) on daily sales, y (in RM). A random sample of six stalls is taken from several shopping centres. The data obtained are summarised as follows:

$$\sum x = 24\ 400, \sum y = 28\ 368, \sum (x-\bar{x})(y-\bar{y}) = 6780,$$

$$\sum (x-\bar{x})^2 = 186\ 333, \sum (y-\bar{y})^2 = 130\ 110.$$

Calculate the coefficient of determination, and comment on your answer. [4 *marks*]

The study also assesses the impact of the size of a shopping centre on daily sales and finds that the coefficient of determination is 0.674. State whether the size of a stall or the size of a shopping centre is more suitable to be used to predict daily sales. Give a reason for your answer. [2 *marks*]

5 The average prices (in thousand ringgit) and the daily quantities for three models of cars produced by a car manufacturer for years 2004 to 2006 are given as follows:

Year	*Model A*		*Model B*		*Model C*	
	Price	*Quantity*	*Price*	*Quantity*	*Price*	*Quantity*
2004	62	120	46	160	38	100
2005	67	200	47	180	33	90
2006	59	140	48	200	29	70

(*a*) Taking year 2004 as the base year, calculate the producer price indices for years 2005 and 2006. [4 *marks*]

(*b*) Comment on the producer price indices obtained. [1 *mark*]

6 The table below shows the quarterly water consumption (in thousand cubic metres) of a factory from the year 2004 to 2007.

Year	*Water consumption*			
	Quarter 1	*Quarter* 2	*Quarter* 3	*Quarter* 4
2004	72	56	50	69
2005	73	56	49	70
2006	76	57	50	71
2007	77	58	52	72

(*a*) Plot a time series, and comment on the appropriateness of a linear trend. [4 *marks*]

(*b*) Calculate the centred four-quarter moving averages for this time series. [4 *marks*]

(*c*) Calculate the seasonal indices using an additive model. [4 *marks*]

4 Satu kajian dijalankan untuk menaksir impak saiz gerai, x (dalam m^2) terhadap jualan harian, y (dalam RM). Satu sampel rawak enam gerai diambil daripada beberapa pusat beli-belah. Data yang diperoleh adalah diikhtisarkan seperti yang berikut:

$$\sum x = 24\ 400, \sum y = 28\ 368, \sum (x-\bar{x})(y-\bar{y}) = 6780,$$

$$\sum (x-\bar{x})^2 = 186\ 333, \sum (y-\bar{y})^2 = 130\ 110.$$

Hitung pekali penentuan, dan berikan ulasan tentang jawapan anda. [4 *markah*]

Kajian itu juga menaksir impak saiz pusat beli-belah terhadap jualan harian dan mendapati bahawa pekali penentuan ialah 0.674. Nyatakan sama ada saiz gerai atau saiz pusat beli-belah lebih sesuai digunakan untuk meramalkan jualan harian. Berikan satu sebab bagi jawapan anda. [2 *markah*]

5 Harga purata (dalam ribu ringgit) dan kuantiti harian bagi tiga model kereta yang dihasilkan oleh sebuah pengilang kereta bagi tahun 2004 hingga 2006 diberikan seperti yang berikut:

Tahun	*Model A*		*Model B*		*Model C*	
	Harga	*Kuantiti*	*Harga*	*Kuantiti*	*Harga*	*Kuantiti*
2004	62	120	46	160	38	100
2005	67	200	47	180	33	90
2006	59	140	48	200	29	70

(*a*) Dengan mengambil tahun 2004 sebagai tahun asas, hitung indeks harga pengeluar bagi tahun 2005 dan 2006. [4 *markah*]

(*b*) Berikan ulasan tentang indeks harga pengeluar yang diperoleh. [1 *markah*]

6 Jadual dibawah menunjukkan penggunaan air (dalam ribu meter padu) suku tahunan sebuah kilang dari tahun 2004 hingga 2007.

Tahun	*Penggunaan air*			
	Sukuan 1	*Sukuan* 2	*Sukuan* 3	*Sukuan* 4
2004	72	56	50	69
2005	73	56	49	70
2006	76	57	50	71
2007	77	58	52	72

(*a*) Plot satu siri masa, dan berikan ulasan tentang kesesuaian trend linear. [4 *markah*]

(*b*) Hitung purata bergerak empat-sukuan berpusat bagi siri masa ini. [4 *markah*]

(*c*) Hitung indeks bermusim dengan menggunakan satu model berdaya tambah. [4 *markah*]

Section B [15 *marks*]

Answer any **one** *question in this section.*

7 An insurance company finds that 8% of the people who are insured with the company against a certain risk make claims within a year.

(*a*) Calculate the probability that at least one out of two randomly selected people who are insured against the risk makes a claim within a year. [3 *marks*]

(*b*) Calculate the probability that at least 3 out of 60 randomly selected people who are insured against the risk make claims within a year. Comment on the validity of the probability model you use. [6 *marks*]

(*c*) Using an approximate probability distribution, estimate the probability that at least 12 out of 100 randomly selected people who are insured against the risk make claims within a year. Justify your use of this approximation. [6 *marks*]

8 The table below shows the values of the variable y corresponding to eight accurately specified values of the variable x.

x	5	7	9	11	13	15	17	19
y	8.5	15.0	27.5	35.0	28.0	37.0	46.0	37.0

(*a*) Plot a scatter diagram for the data. [2 *marks*]

(*b*) Calculate the Pearson correlation coefficient r between x and y. [5 *marks*]

(*c*) Comment on your value of r with respect to the scatter diagram in (*a*). [1 *mark*]

(*d*) Find the equation of the least squares regression line in the form $y = a + bx$, where a and b are constants. [4 *marks*]

(*e*) Estimate the values of y for $x = 0, 10, 20, 30$, where appropriate. Give a reason why it is inappropriate to estimate the values of y for the other given values of x. [3 *marks*]

Bahagian B [15 *markah*]

Jawab mana-mana **satu** *soalan dalam bahagian ini.*

7 Satu syarikat insurans mendapati bahawa 8% orang yang membeli insurans dengan syarikat ini terhadap risiko tertentu membuat tuntutan dalam jangka masa satu tahun.

(*a*) Hitung kebarangkalian bahawa sekurang-kurangnya seorang daripada dua orang yang dipilih secara rawak yang diinsurankan terhadap risiko itu membuat tuntutan dalam jangka masa satu tahun. [3 *markah*]

(*b*) Hitung kebarangkalian bahawa sekurang-kurangnya tiga orang daripada 60 orang yang dipilih secara rawak yang diinsurankan terhadap risiko itu membuat tuntutan dalam jangka masa satu tahun. Berikan ulasan tentang kesahan model kebarangkalian yang anda gunakan. [6 *markah*]

(*c*) Dengan menggunakan taburan kebarangkalian hampiran, anggar kebarangkalian bahawa sekurang-kurangnya 12 orang daripada 100 orang yang dipilih secara rawak yang diinsurankan terhadap risiko itu membuat tuntutan dalam jangka masa satu tahun. Justifikasikan penghampiran yang anda gunakan ini. [6 *markah*]

8 Jadual di bawah menunjukkan nilai pembolehubah y yang sepadan dengan lapan nilai pembolehubah x yang ditentukan secara jitu.

x	5	7	9	11	13	15	17	19
y	8.5	15.0	27.5	35.0	28.0	37.0	46.0	37.0

(*a*) Plot satu gambar rajah serakan bagi data itu. [2 *markah*]

(*b*) Hitung pekali korelasi Pearson r antara x dengan y. [5 *markah*]

(*c*) Berikan ulasan tentang nilai r terhadap gambar rajah serakan dalam (*a*). [1 *markah*]

(*d*) Cari persamaan garis regresi kuasa dua terkecil dalam bentuk $y = a + bx$, dengan a dan b pemalar. [4 *markah*]

(*e*) Anggar nilai y bagi $x = 0, 10, 20, 30$, di mana sesuai. Berikan satu sebab mengapa tidak sesuai untuk menganggar nilai y bagi nilai x lain yang diberikan. [3 *markah*]

MATHEMATICAL FORMULAE (RUMUS MATEMATIK)

Probability distributions (*Taburan kebarangkalian*)

Binomial distribution (Taburan binomial)

$$\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n$$

Poisson distribution (Taburan Poisson)

$$\mathrm{P}(X = x) = \frac{\mathrm{e}^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, \ldots$$

Correlation and regression (*Korelasi dan regresi*)

Pearson correlation coefficient (Pekali korelasi Pearson)

$$r = \frac{\sum xy - \frac{(\sum x)(\sum y)}{n}}{\sqrt{\sum x^2 - \frac{(\sum x)^2}{n}}\sqrt{\sum y^2 - \frac{(\sum y)^2}{n}}}$$

Spearman rank correlation coefficient (Pekali korelasi pangkat Spearman)

$$r = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)}$$

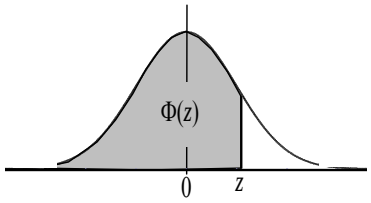
Least squares regression line (Garis regresi kuasa dua terkecil)

$$y = a + bx, \quad b = \frac{\sum xy - \frac{(\sum x)(\sum y)}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}}, \quad a = \bar{y} - b\bar{x}$$

THE NORMAL DISTRIBUTION FUNCTION (FUNGSI TABURAN NORMAL)

If Z has a normal distribution with mean 0 and variance 1, then for each value of z, the tabulated value of $\Phi(z)$ is such that $\Phi(z) = P(Z \leqslant z)$. For negative values of z, use $\Phi(-z) = 1 - \Phi(z)$.

Jika Z mempunyai taburan normal dengan min 0 dan varians 1, maka bagi setiap nilai z, nilai terjadual $\Phi(z)$ adalah sebegitu rupa sehinggakan $\Phi(z) = P(Z \leqslant z)$. Bagi nilai negatif z, gunakan $\Phi(-z) = 1 - \Phi(z)$.



z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
											ADD (TAMBAH)								
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	31	35
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	8	11	15	19	23	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	21	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	6	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	6	9	11	14	17	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	11	14	16	18	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	19
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	5	7	9	11	13	15	16
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	3	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	2	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	1	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

Critical values for the normal distribution (Nilai genting bagi taburan normal)

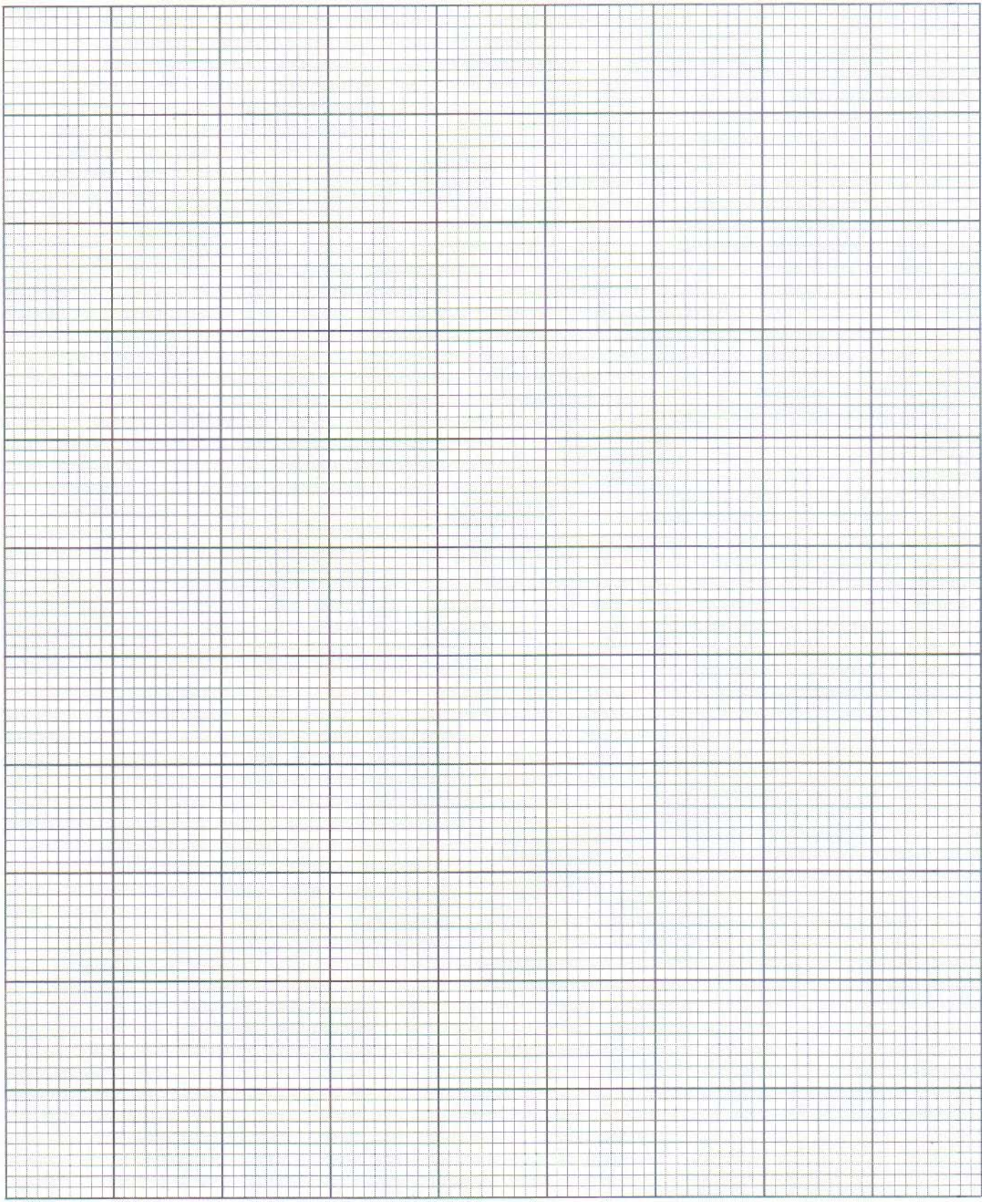
If Z has a normal distribution with mean 0 and variance 1, then for each value of p, the tabulated value of z is such that $P(Z \leqslant z) = p$.

Jika Z mempunyai taburan normal dengan min 0 dan varians 1, maka bagi setiap nilai p, nilai terjadual z adalah sebegitu rupa sehinggakan $P(Z \leqslant z) = p$.

p	0.75	0.9	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

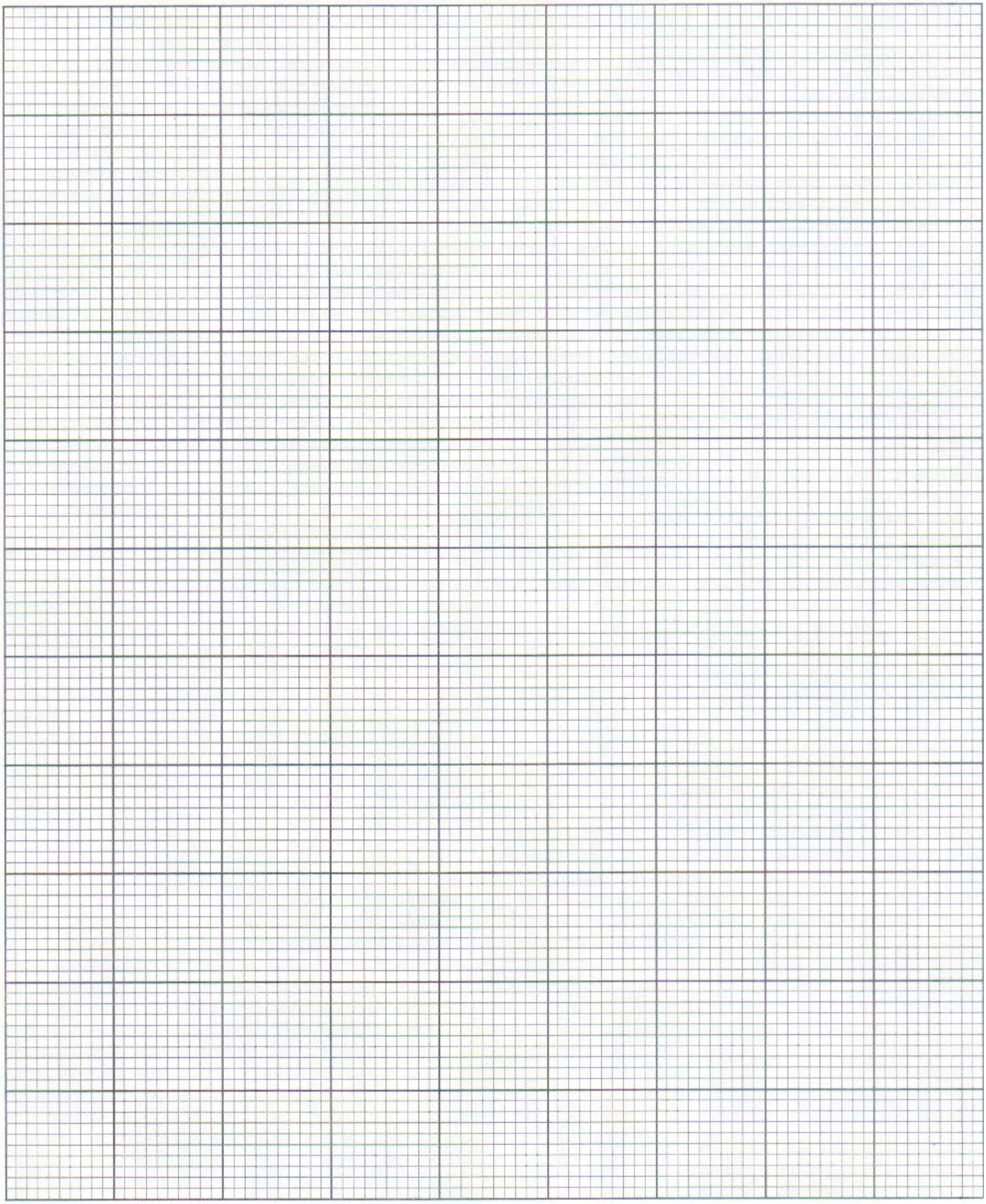
BLANK PAGE

Identity card number:..... Centre number/index number:.....
(Nombor kad pengenalan) (Nombor pusat/angka giliran)



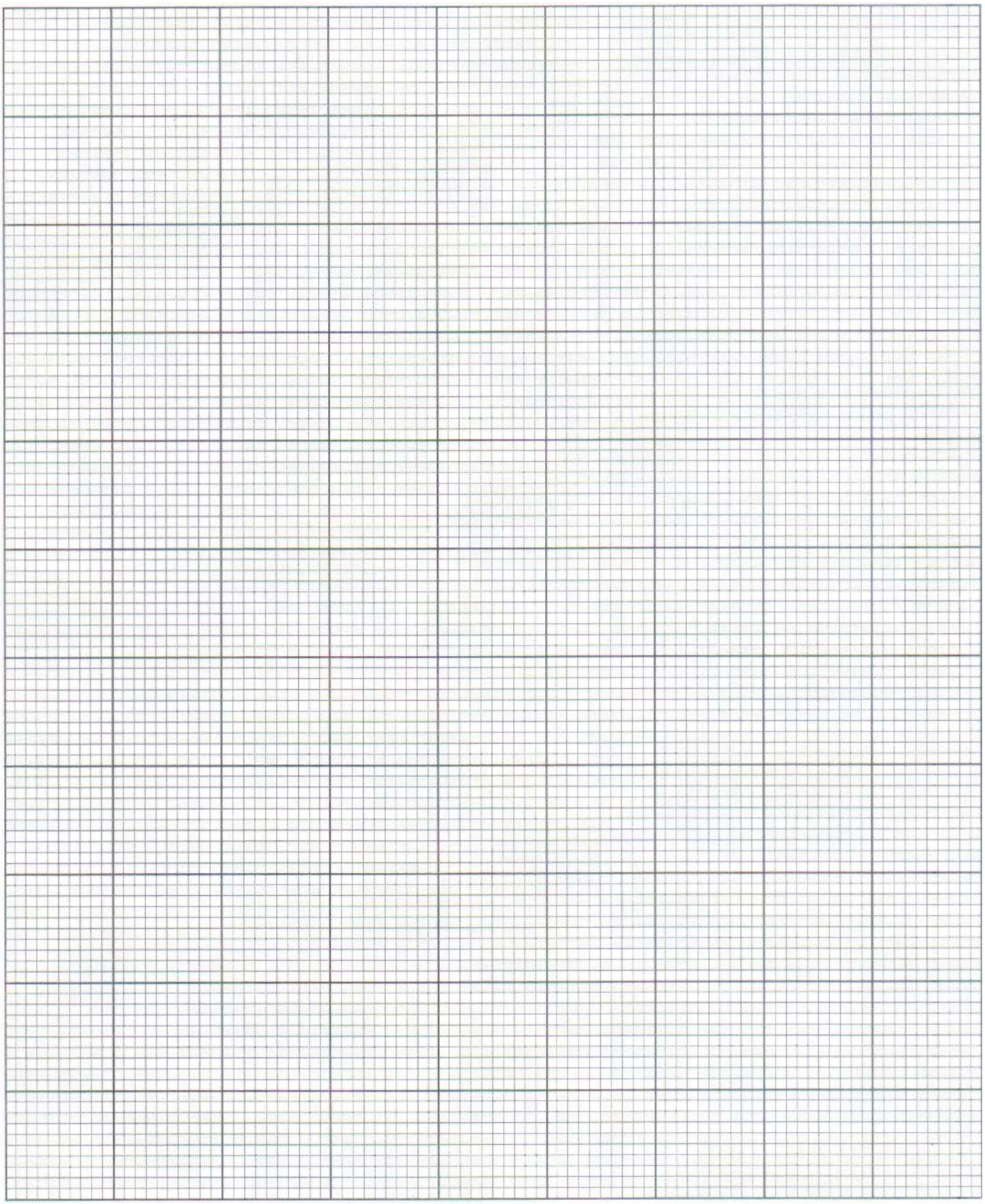
Please tear off along the perforated line.
(Sila koyakkan di sepanjang garis putus-putus ini.)

Identity card number:..... Centre number/index number:.....
(Nombor kad pengenalan) (Nombor pusat/angka giliran)



950/2

Identity card number:..... Centre number/index number:.....
(Nombor kad pengenalan) (Nombor pusat/angka giliran)



Please tear off along the perforated line.
(Sila koyakkan di sepanjang garis putus-putus ini.)

950/2

SPECIMEN PAPER

950/3 STPM

MATHEMATICS (M) (MATEMATIK (M))

PAPER 3 (KERTAS 3)

One and a half hours (Satu jam setengah)

MAJLIS PEPERIKSAAN MALAYSIA
(MALAYSIAN EXAMINATIONS COUNCIL)

SIJIL TINGGI PERSEKOLAHAN MALAYSIA
(MALAYSIA HIGHER SCHOOL CERTIFICATE)

Instruction to candidates:

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Answer **all** *questions in Section A and any* **one** *question in Section B. Answers may be written in either English or Bahasa Malaysia.*

All necessary working should be shown clearly.

Scientific calculators may be used. Programmable and graphic display calculators are prohibited.

A list of mathematical formulae and graph papers are provided on pages of this question paper.

Arahan kepada calon:

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU UNTUK BERBUAT DEMIKIAN.

Jawab **semua** *soalan dalam Bahagian A dan mana-mana* **satu** *soalan dalam Bahagian B. Jawapan boleh ditulis dalam bahasa Inggeris atau Bahasa Malaysia.*

Semua kerja yang perlu hendaklah ditunjukkan dengan jelas.

Kalkulator sainstifik boleh digunakan. Kalkulator boleh atur cara dan kalkulator paparan grafik tidak dibenarkan.

Senarai rumus matematik dan kertas graf dibekalkan pada halaman dalam kertas soalan ini.

This question paper consists of printed pages and blank page.
(Kertas soalan ini terdiri daripada halaman bercetak dan halaman kosong.)

Section A [45 *marks*]

Answer **all** *questions in this section.*

1 A car which costs RM80 000 is bought with a down payment of RM8000 and a loan of RM72 000. The loan is amortised with 60 monthly payments at an annual interest rate of 3.6%, compounded monthly. The car depreciates at the rate of 10% per year.

(*a*) Calculate the monthly payment needed to amortise the loan. [3 *marks*]

(*b*) Calculate the total amount of interest paid over 5 years. [2 *marks*]

(*c*) Using the reducing balance depreciation method, find the book value of the car when the loan is paid off. [3 *marks*]

2 The demand for a campus newspaper is 2300 copies each week if the newspaper is free of charge; it drops to 1300 copies each week if the newspaper cost RM0.25 per copy. The university is prepared to supply 500 copies each week free of charge or alternatively 2000 copies each week at RM0.30 per copy.

(*a*) Find the linear demand and supply functions. [4 *marks*]

(*b*) Determine the price at which the newspaper is to be sold so that there is neither a shortage nor a surplus. [2 *marks*]

3 A company produces two types of lamps, *A* and *B*, which are made of three types of material: iron frame, electrical component and plastic component. Each lamp *A* requires 1 unit of iron frame, 2 units of electrical component and 3 units of plastic component, whereas each lamp *B* requires 3 units of iron frame, 2 units of electrical component and 1 unit of plastic component. The company has 300 000 units of iron frame, 300 000 units of electrical component and 400 000 units of plastic component in stock. The profits made from each lamp *A* and lamp *B* are RM15.00 and RM20.00 respectively.

(*a*) Formulate a linear programming problem to maximise profit subject to the constraints. [4 *marks*]

(*b*) Using the graphical method, determine the number of lamp *A* and the number of lamp *B* which give the maximum profit, and find this maximum profit. [8 *marks*]

4 A project on setting a student registration system of a college involves seven activities. The activities, their preceding activities and duration (in days) are listed as follows:

Activity	*Preceding activities*	*Duration*
A	–	4
B	–	2
C	–	3
D	*A*	8
E	*B*	6
F	*C*	3
G	*D* , *E*	4

Determine the earliest start time and the latest finish time for each activity, and hence, find the minimum duration for the project to be completed. [6 *marks*]

Bahagian A [45 *markah*]

Jawab **semua** *soalan dalam bahagian ini*.

1 Sebuah kereta berharga RM80 000 dibeli dengan bayaran pendahuluan RM8000 dan pinjaman RM72 000. Pinjaman itu dilunaskan dengan bayaran bulanan 60 bulan pada kadar faedah tahunan 3.6%, dikompaun secara bulanan. Nilai kereta itu susut pada kadar 10% per tahun.

(*a*) Hitung bayaran balik bulanan yang diperlukan untuk melunaskan pinjaman itu. [3 *markah*]

(*b*) Hitung jumlah amaun faedah yang dibayar selama 5 tahun. [2 *markah*]

(*c*) Dengan menggunakan kaedah susut nilai baki berkurangan, cari nilai buku kereta itu apabila pinjaman habis diselesaikan. [3 *markah*]

2 Permintaan akhbar kampus ialah 2300 naskhah setiap minggu jika akhbar itu percuma; permintaan itu menurun kepada 1300 naskhah setiap minggu jika harga akhbar itu ialah RM0.25 per naskhah. Universiti itu bersedia untuk menawarkan 500 naskhah percuma setiap minggu atau sebagai alternatif 2000 naskhah setiap minggu dengan harga RM0.30 per naskhah.

(*a*) Cari fungsi permintaan dan fungsi penawaran linear. [4 *markah*]

(*b*) Tentukan harga surat khabar yang perlu dijual supaya tidak berlaku kurangan atau lebihan. [2 *markah*]

3 Satu syarikat menghasilkan dua jenis lampu, *A* dan *B*, yang diperbuat daripada tiga jenis bahan: bingkai besi, komponen elektrik dan komponen plastik. Setiap lampu *A* memerlukan 1 unit bingkai besi, 2 unit komponen elektrik dan 3 unit komponen plastik, manakala setiap lampu *B* memerlukan 3 unit bingkai besi, 2 unit komponen elektrik dan 1 unit komponen plastik. Syarikat itu mempunyai 300 000 unit bingkai besi, 300 000 unit komponen elektrik dan 400 000 unit komponen plastik dalam stok. Keuntungan yang diperoleh daripada setiap lampu *A* dan lampu *B* masing-masing ialah RM15.00 dan RM20.00.

(*a*) Rumuskan satu masalah pengaturcaraan linear untuk memaksimumkan keuntungan tertakluk kepada kekangan itu. [4 *markah*]

(*b*) Dengan menggunakan kaedah bergraf, tentukan bilangan lampu *A* dan bilangan lampu *B* yang memberikan keuntungan maksimum, dan cari keuntungan maksimum ini. [8 *markah*]

4 Satu projek tentang pembangunan sistem pendaftaran pelajar di sebuah kolej melibatkan tujuh kegiatan. Kegiatan, kegiatan sebelumnya dan tempoh (dalam hari) disenaraikan seperti yang berikut:

Kegiatan	*Kegiatan sebelumnya*	*Tempoh*
A	–	4
B	–	2
C	–	3
D	*A*	8
E	*B*	6
F	*C*	3
G	*D* , *E*	4

Tentukan masa mula terawal dan masa tamat terlewat bagi setiap kegiatan, dan dengan yang demikian, cari tempoh minimum bagi menyempurnakan projek itu. [6 *markah*]

5 A retailer expects to sell 500 units of an electronic component per year. The cost per unit is RM30 and the ordering cost is RM50 per order. The holding cost per unit of this product is RM3 per year. The replacement is instantaneous and no shortages are allowed.

(*a*) Find the optimum economic lot size. [2 *marks*]

(*b*) Determine the number of orders per year. [3 *marks*]

(*c*) Calculate the total cost per year. [4 *marks*]

6 A two-person zero-sum game for players *A* and *B* is represented by the pay-off matrix

		B		
		I	II	III
A	I	2	1	3
	II	3	–5	4
	III	0	1	5

(*a*) Show that the game has a stable solution. [3 *marks*]

(*b*) State the value of the game. [1 *mark*]

5 Seorang peruncit menjangka untuk menjual 500 unit sejenis komponen elektronik per tahun. Kos per unit ialah RM30 dan kos pesanan ialah RM50 per pesanan. Kos pegangan per unit produk ini ialah RM3 per tahun. Penggantian adalah serta merta dan tiada kurangan yang dibenarkan.

(*a*) Cari saiz lot ekonomi optimum. [2 *markah*]

(*b*) Tentukan bilangan pesanan per tahun. [3 *markah*]

(*c*) Hitung jumlah kos per tahun. [4 *markah*]

6 Satu permainan *two-person zero-sum* bagi pemain *A* dan *B* diwakilkan oleh matriks timbal-balik

		B		
		I	II	III
A	I	2	1	3
	II	3	–5	4
	III	0	1	5

(*a*) Tunjukkan bahawa permainan itu mempunyai satu penyelesaian stabil. [3 *markah*]

(*b*) Nyatakan nilai permainan itu. [1 *markah*]

Section B [15 *marks*]

Answer any **one** *question in this section.*

7 The number of units, x, demanded for a certain product depends on the unit price, p (in RM), and is given by

$$x = \sqrt{120\,000 - 2p},$$

where $0 < p < 60\,000$. The cost C (in RM) of producing x units is given by

$$C = 35\,000x - 0.1x^3,$$

where $0 < x \leqslant 300$.

(*a*) Find the revenue in terms of x. [3 *marks*]

(*b*) Determine the demand and the unit price for which the revenue is maximum and find the maximum revenue. [6 *marks*]

(*c*) Calculate the average cost per unit at the production level in (*b*). [2 *marks*]

(*d*) Determine the production levels at which the profit is increasing. [4 *marks*]

8 Two computer companies X and Y are the only sale and service providers in a region. Both companies advertise their sales and services through radio (I), television (II) and newspaper (III). Depending on the effectiveness of each advertising campaign, a company can capture a portion of the customers from the other. The following pay-off matrix summarises the percentages of the customers captured or lost by company X.

		Y		
		I	II	III
X	I	5	–15	5
	II	–5	10	–20
	III	15	5	–10

(*a*) State a reason why company Y should not use the radio as a medium of advertising. [1 *mark*]

(*b*) Use a graphical method to determine the optimal strategy of company Y, and hence, find the percentage gain of the company. [10 *marks*]

(*c*) Determine the optimal strategy of company X. [4 *marks*]

Bahagian B [15 *markah*]

Jawab mana-mana **satu** *soalan dalam bahagian ini.*

7 Bilangan unit, x, yang diminta bagi satu produk tertentu bergantung pada harga unit, p (dalam RM), dan diberikan oleh

$$x = \sqrt{120\,000 - 2p},$$

dengan $0 < p < 60\,000$. Kos C (dalam RM) untuk menghasilkan x unit diberikan oleh

$$C = 35\,000x - 0.1x^3,$$

dengan $0 < x \leqslant 300$.

(*a*) Cari hasil dalam sebutan x. [3 *markah*]

(*b*) Tentukan permintaan dan harga unit supaya hasil adalah maksimum dan cari hasil maksimum itu. [6 *markah*]

(*c*) Hitung kos purata per unit pada aras pengeluaran dalam (*b*). [2 *markah*]

(*d*) Tentukan aras pengeluaran supaya keuntungan menokok. [4 *markah*]

8 Dua syarikat komputer *X* dan *Y* sahaja yang menawarkan jualan dan perkhidmatan di satu rantau. Kedua-dua syarikat mengiklankan jualan dan perkhidmatan mereka melalui radio (I), televisyen (II), dan surat khabar (III). Bergantung kepada keberkesanan setiap kempen pengiklanan, satu syarikat boleh menarik sebahagian daripada pelanggan daripada syarikat pesaing. Matriks timbal-balik yang berikut mengikhtisarkan peratusan pelanggan yang ditarik atau hilang oleh syarikat *X*.

		Y		
		I	II	III
X	I	5	–15	5
	II	–5	10	–20
	III	15	5	–10

(*a*) Nyatakan satu sebab mengapa syarikat *Y* tidak patut menggunakan radio sebagai perantara pengiklanan. [1 *markah*]

(*b*) Gunakan kaedah bergraf untuk menentukan strategi optimum syarikat *Y*, dan dengan yang demikian, cari peratusan keuntungan syarikat itu. [10 *markah*]

(*c*) Tentukan strategi optimum syarikat *X*. [4 *markah*]

MATHEMATICAL FORMULAE (RUMUS MATEMATIK)

Interest and annuity (*Faedah dan anuiti*)

Effective interest rate (Kadar faedah berkesan)

$$r_{\text{eff}} = (1+i)^n - 1$$

Future value of an annuity (Nilai hadapan anuiti)

$$A = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Present value of an annuity (Nilai kini anuiti)

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Inventory models (*Model inventori*)

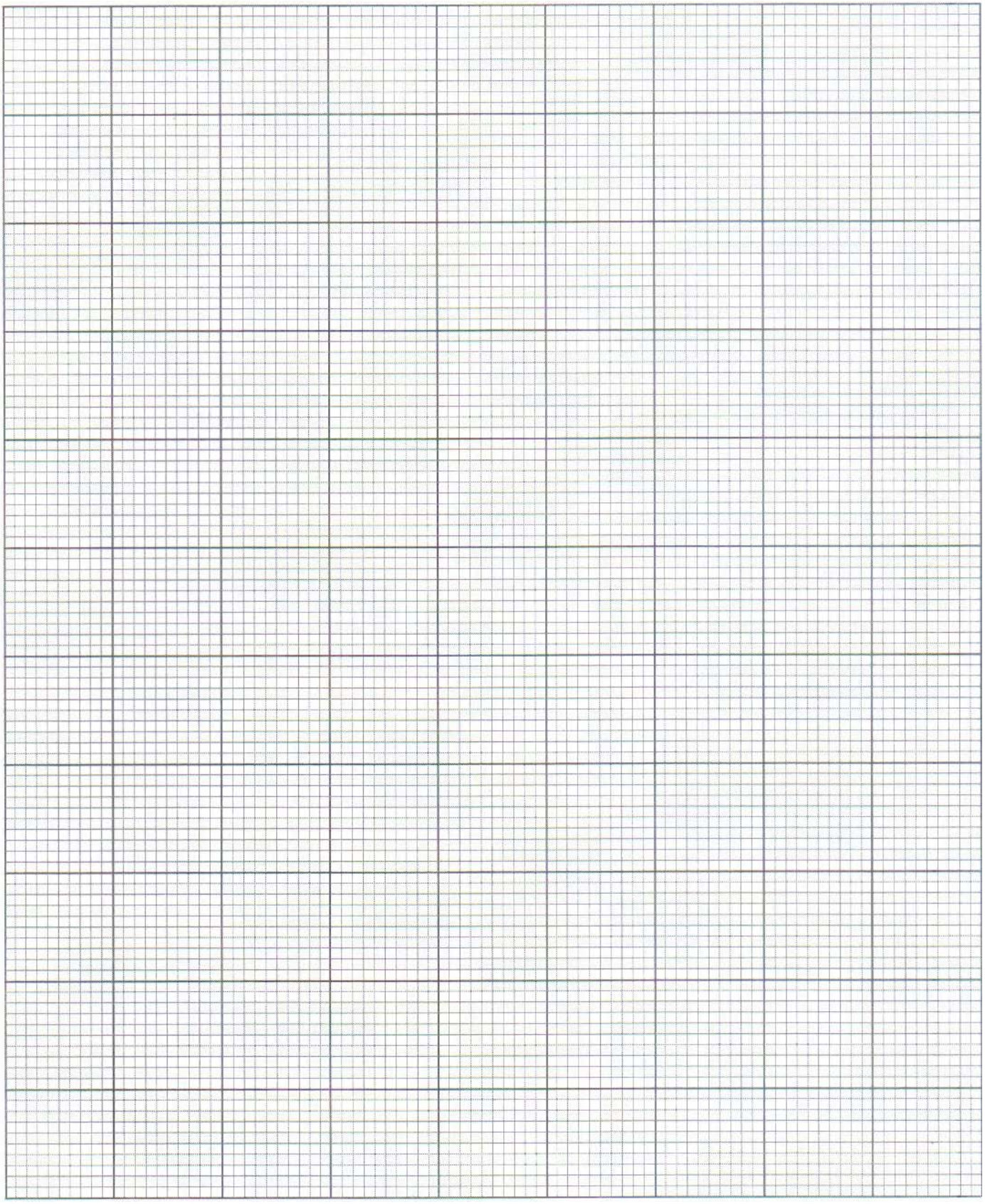
For basic EOQ model (Bagi model EOQ asas)

$$Q^* = \sqrt{\frac{2DC_0}{C_h}}$$

For EOQ model with planned shortages (Bagi model EOQ dengan kurangan terancang)

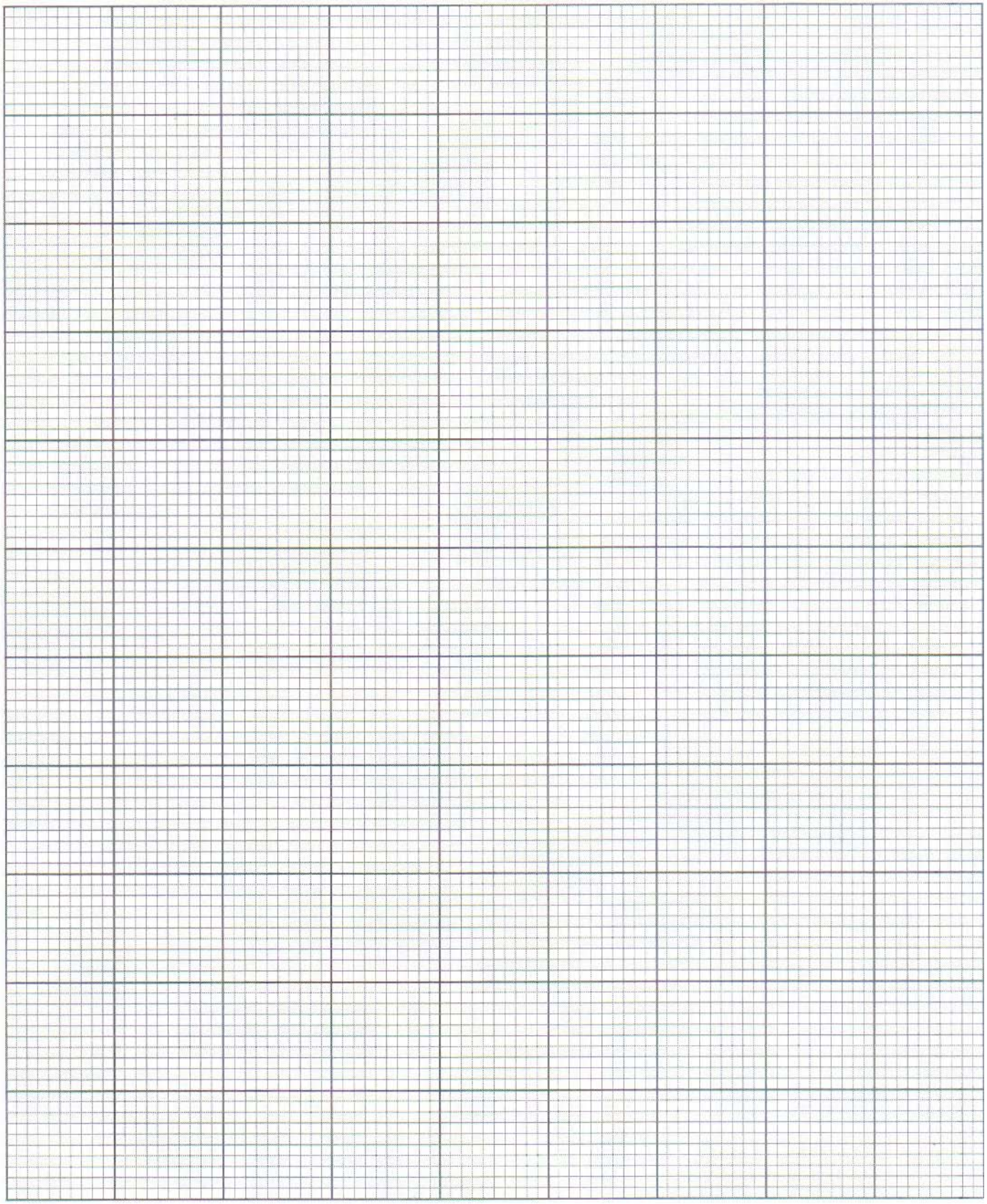
$$Q^* = \sqrt{\frac{2DC_0}{C_h}\left(\frac{C_h + C_b}{C_b}\right)}$$

Identity card number:..... Centre number /index number:.....
(Nombor kad pengenalan) (Nombor pusat/angka giliran)



Please tear off along the perforated line.
(Sila koyakkan di sepanjang garis putus-putus ini.)

Identity card number:..... Centre number /index number:.....
(Nombor kad pengenalan) (Nombor pusat/angka giliran)



950/3

SPECIMEN ASSIGNMENT

950/4 **STPM**

MATHEMATICS (M) (MATEMATIK (M))

PAPER 4 (KERTAS 4)

MAJLIS PEPERIKSAAN MALAYSIA
(MALAYSIAN EXAMINATIONS COUNCIL)

SIJIL TINGGI PERSEKOLAHAN MALAYSIA
(MALAYSIA HIGHER SCHOOL CERTIFICATE)

Many projects require proper planning, management and resource scheduling in order that they can be completed in time. A project is modelled by a network.

The first step in the modelling of a project is to break it down into a set of subprojects called activities and then look at how each activity precedes or affects other activities of the project.

The table below shows the activities and their preceding activities in the building of a house and the duration and number of workers required for each activity.

Activity	*Description*	*Preceding activity*	*Duration* (*weeks*)	*Number of workers*
A	Prepare the area and foundation	-	5	3
B	Prepare the pillars and frame	*A*	8	3
C	Fixing the roof	*B*	5	2
D	Erecting the walls	*B*	7	3
E	Exterior plumbing	*D*	2	4
F	Interior plumbing	*E*	3	4
G	Electrical work	*D*	4	3
H	Plastering the walls	*F, G*	7	4
I	Fixing the ceiling	*C*	4	4
J	Painting the house	*H,I*	5	3
K	Flooring	*H*	4	2
L	Interior fixture	*K*	4	2
M	Exterior fixture	*J*	3	2

1. (*a*) Draw an activity-on-node network for the project.

   (*b*) List down all possible paths and possible durations of the project.

   (*c*) Deduce the critical activities, and state the minimum time for the completion of the project.

2. (*a*) Construct a table to include the earliest start time (EST), earliest finish time (EFT), latest start time (LST) and latest finish time (LFT) for each activities.

   (*b*) Determine the total float for each activity.

   (*c*) Identify the critical activities, and determine the minimum time for the completion of the project.

3. Compare the methods used in 1 and 2.

4. (*a*) Draw a Gantt chart for the activities, and hence, draw a resource histogram.

   (*b*) Analyse the number of workers required to complete the project in the minimum time.