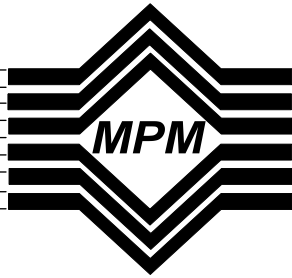


STPM/S(E)954

MAJLIS PEPERIKSAAN MALAYSIA
(MALAYSIAN EXAMINATIONS COUNCIL)



PEPERIKSAAN
SIJIL TINGGI PERSEKOLAHAN MALAYSIA
(MALAYSIA HIGHER SCHOOL CERTIFICATE EXAMINATION)

MATHEMATICS (T)
Syllabus and Specimen Papers

This syllabus applies for the 2012/2013 session and thereafter until further notice.

NATIONAL EDUCATION PHILOSOPHY

“Education in Malaysia is an on-going effort towards further developing the potential of individuals in a holistic and integrated manner, so as to produce individuals who are intellectually, spiritually, emotionally and physically balanced and harmonious, based on a belief in and devotion to God. Such effort is designed to produce Malaysian citizens who are knowledgeable and competent, who possess high moral standards, and who are responsible and capable of achieving a high level of personal well-being as well as being able to contribute to the betterment of the family, the society and the nation at large.”

FOREWORD

This revised Mathematics (T) syllabus is designed to replace the existing syllabus which has been in use since the 2002 STPM examination. This new syllabus will be enforced in 2012 and the first examination will also be held the same year. The revision of the syllabus takes into account the changes made by the Malaysian Examinations Council (MEC) to the existing STPM examination. Through the new system, the form six study will be divided into three terms, and candidates will sit for an examination at the end of each term. The new syllabus fulfils the requirements of this new system. The main objective of introducing the new examination system is to enhance the teaching and learning orientation of form six so as to be in line with the orientation of teaching and learning in colleges and universities.

The Mathematics (T) syllabus is designed to provide a framework for a pre-university course that enables candidates to develop the understanding of mathematical concepts and mathematical thinking, and acquire skills in problem solving and the applications of mathematics related to science and technology. The assessment tools of this syllabus consist of written papers and coursework. Coursework offers opportunities for candidates to conduct mathematical investigation and mathematical modelling that enhance their understanding of mathematical processes and applications and provide a platform for them to develop soft skills.

The syllabus contains topics, teaching periods, learning outcomes, examination format, grade description and specimen papers.

The design of this syllabus was undertaken by a committee chaired by Professor Dr. Abu Osman bin Md Tap from International Islamic University Malaysia. Other committee members consist of university lecturers, representatives from the Curriculum Development Division, Ministry of Education Malaysia, and experienced teachers who are teaching Mathematics. On behalf of MEC, I would like to thank the committee for their commitment and invaluable contribution. It is hoped that this syllabus will be a guide for teachers and candidates in the teaching and learning process.

Chief Executive
Malaysian Examinations Council

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SYLLABUS
954 MATHEMATICS (T)
[May not be taken with 950 Mathematics (M)]

Aims

The Mathematics (T) syllabus is designed to provide a framework for a pre-university course that enables candidates to develop the understanding of mathematical concepts and mathematical thinking, and acquire skills in problem solving and the applications of mathematics related to science and technology.

Objectives

The objectives of the syllabus are to enable candidates to:

- (a) use mathematical concepts, terminology and notation;
- (b) display and interpret mathematical information in tabular, diagrammatic and graphical forms;
- (c) identify mathematical patterns and structures in a variety of situations;
- (d) use appropriate mathematical models in different contexts;
- (e) apply mathematical principles and techniques in solving problems;
- (f) carry out calculations and approximations to an appropriate degree of accuracy;
- (g) interpret the significance and reasonableness of results;
- (h) present mathematical explanations, arguments and conclusions.

FIRST TERM: ALGEBRA AND GEOMETRY

<i>Topic</i>	<i>Teaching Period</i>	<i>Learning Outcome</i>
1 Functions	28	Candidates should be able to:
1.1 Functions	6	(a) state the domain and range of a function, and find composite functions; (b) determine whether a function is one-to-one, and find the inverse of a one-to-one function; (c) sketch the graphs of simple functions, including piecewise-defined functions;
1.2 Polynomial and rational functions	8	(d) use the factor theorem and the remainder theorem; (e) solve polynomial and rational equations and inequalities; (f) solve equations and inequalities involving modulus signs in simple cases; (g) decompose a rational expression into partial fractions in cases where the denominator has two distinct linear factors, or a linear factor and a prime quadratic factor;
1.3 Exponential and logarithmic functions	6	(h) relate exponential and logarithmic functions, algebraically and graphically; (i) use the properties of exponents and logarithms; (j) solve equations and inequalities involving exponential or logarithmic expressions;
1.4 Trigonometric functions	8	(k) relate the periodicity and symmetries of the sine, cosine and tangent functions to their graphs, and identify the inverse sine, inverse cosine and inverse tangent functions and their graphs; (l) use basic trigonometric identities and the formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$, including $\sin 2A$, $\cos 2A$ and $\tan 2A$; (m) express $a \sin \theta + b \cos \theta$ in the forms $r \sin(\theta \pm \alpha)$ and $r \cos(\theta \pm \alpha)$; (n) find the solutions, within specified intervals, of trigonometric equations and inequalities.

<i>Topic</i>	<i>Teaching Period</i>	<i>Learning Outcome</i>
2 Sequences and Series	18	Candidates should be able to:
2.1 Sequences	4	(a) use an explicit formula and a recursive formula for a sequence; (b) find the limit of a convergent sequence;
2.2 Series	8	(c) use the formulae for the n th term and for the sum of the first n terms of an arithmetic series and of a geometric series; (d) identify the condition for the convergence of a geometric series, and use the formula for the sum of a convergent geometric series; (e) use the method of differences to find the n th partial sum of a series, and deduce the sum of the series in the case when it is convergent;
2.3 Binomial expansions	6	(f) expand $(a + b)^n$, where $n \in \mathbb{Z}^+$; (g) expand $(1 + x)^n$, where $n \in \mathbb{Q}$, and identify the condition $ x < 1$ for the validity of this expansion; (h) use binomial expansions in approximations.
3 Matrices	16	Candidates should be able to:
3.1 Matrices	10	(a) identify null, identity, diagonal, triangular and symmetric matrices; (b) use the conditions for the equality of two matrices; (c) perform scalar multiplication, addition, subtraction and multiplication of matrices with at most three rows and three columns; (d) use the properties of matrix operations; (e) find the inverse of a non-singular matrix using elementary row operations; (f) evaluate the determinant of a matrix; (g) use the properties of determinants;

<i>Topic</i>	<i>Teaching Period</i>	<i>Learning Outcome</i>
3.2 Systems of linear equations	6	<p>(h) reduce an augmented matrix to row-echelon form, and determine whether a system of linear equations has a unique solution, infinitely many solutions or no solution;</p> <p>(i) apply the Gaussian elimination to solve a system of linear equations;</p> <p>(j) find the unique solution of a system of linear equations using the inverse of a matrix.</p>
4 Complex Numbers	12	<p>Candidates should be able to:</p> <p>(a) identify the real and imaginary parts of a complex number;</p> <p>(b) use the conditions for the equality of two complex numbers;</p> <p>(c) find the modulus and argument of a complex number in cartesian form and express the complex number in polar form;</p> <p>(d) represent a complex number geometrically by means of an Argand diagram;</p> <p>(e) find the complex roots of a polynomial equation with real coefficients;</p> <p>(f) perform elementary operations on two complex numbers expressed in cartesian form;</p> <p>(g) perform multiplication and division of two complex numbers expressed in polar form;</p> <p>(h) use de Moivre's theorem to find the powers and roots of a complex number.</p>
5 Analytic Geometry	14	<p>Candidates should be able to:</p> <p>(a) transform a given equation of a conic into the standard form;</p> <p>(b) find the vertex, focus and directrix of a parabola;</p> <p>(c) find the vertices, centre and foci of an ellipse;</p> <p>(d) find the vertices, centre, foci and asymptotes of a hyperbola;</p> <p>(e) find the equations of parabolas, ellipses and hyperbolas satisfying prescribed conditions (excluding eccentricity);</p>

<i>Topic</i>	<i>Teaching Period</i>	<i>Learning Outcome</i>
6 Vectors	20	(f) sketch conics; (g) find the cartesian equation of a conic defined by parametric equations; (h) use the parametric equations of conics. Candidates should be able to:
6.1 Vectors in two and three dimensions	8	(a) use unit vectors and position vectors; (b) perform scalar multiplication, addition and subtraction of vectors; (c) find the scalar product of two vectors, and determine the angle between two vectors; (d) find the vector product of two vectors, and determine the area a parallelogram and of a triangle;
6.2 Vector geometry	12	(e) find and use the vector and cartesian equations of lines; (f) find and use the vector and cartesian equations of planes; (g) calculate the angle between two lines, between a line and a plane, and between two planes; (h) find the point of intersection of two lines, and of a line and a plane; (i) find the line of intersection of two planes.

SECOND TERM: CALCULUS

<i>Topic</i>	<i>Teaching Period</i>	<i>Learning Outcome</i>
7 Limits and Continuity	12	Candidates should be able to:
7.1 Limits	6	(a) determine the existence and values of the left-hand limit, right-hand limit and limit of a function; (b) use the properties of limits;
7.2 Continuity	6	(c) determine the continuity of a function at a point and on an interval; (d) use the intermediate value theorem.
8 Differentiation	28	Candidates should be able to:
8.1 Derivatives	12	(a) identify the derivative of a function as a limit; (b) find the derivatives of x^n ($n \in \mathbb{Q}$), e^x , $\ln x$, $\sin x$, $\cos x$, $\tan x$, $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$, with constant multiples, sums, differences, products, quotients and composites; (c) perform implicit differentiation; (d) find the first derivatives of functions defined parametrically;
8.2 Applications of differentiation	16	(e) determine where a function is increasing, decreasing, concave upward and concave downward; (f) determine the stationary points, extremum points and points of inflexion; (g) sketch the graphs of functions, including asymptotes parallel to the coordinate axes; (h) find the equations of tangents and normals to curves, including parametric curves; (i) solve problems concerning rates of change, including related rates; (j) solve optimisation problems.
9 Integration	28	Candidates should be able to:
9.1 Indefinite integrals	14	(a) identify integration as the reverse of differentiation; (b) integrate x^n ($n \in \mathbb{Q}$), e^x , $\sin x$, $\cos x$, $\sec^2 x$, with constant multiples, sums and differences;

<i>Topic</i>	<i>Teaching Period</i>	<i>Learning Outcome</i>
9.2 Definite integrals	14	<ul style="list-style-type: none"> (c) integrate rational functions by means of decomposition into partial fractions; (d) use trigonometric identities to facilitate the integration of trigonometric functions; (e) use algebraic and trigonometric substitutions to find integrals; (f) perform integration by parts; (g) identify a definite integral as the area under a curve; (h) use the properties of definite integrals; (i) evaluate definite integrals; (j) calculate the area of a region bounded by a curve (including a parametric curve) and lines parallel to the coordinate axes, or between two curves; (k) calculate volumes of solids of revolution about one of the coordinate axes.
10 Differential Equations	14	<p>Candidates should be able to:</p> <ul style="list-style-type: none"> (a) find the general solution of a first order differential equation with separable variables; (b) find the general solution of a first order linear differential equation by means of an integrating factor; (c) transform, by a given substitution, a first order differential equation into one with separable variables or one which is linear; (d) use a boundary condition to find a particular solution; (e) solve problems, related to science and technology, that can be modelled by differential equations.
11 Maclaurin Series	12	<p>Candidates should be able to:</p> <ul style="list-style-type: none"> (a) find the Maclaurin series for a function and the interval of convergence; (b) use standard series to find the series expansions of the sums, differences, products, quotients and composites of functions;

<i>Topic</i>	<i>Teaching Period</i>	<i>Learning Outcome</i>
12 Numerical Methods	14	(c) perform differentiation and integration of a power series; (d) use series expansions to find the limit of a function. Candidates should be able to:
12.1 Numerical solution of equations	10	(a) locate a root of an equation approximately by means of graphical considerations and by searching for a sign change; (b) use an iterative formula of the form $x_{n+1} = f(x_n)$ to find a root of an equation to a prescribed degree of accuracy; (c) identify an iteration which converges or diverges; (d) use the Newton-Raphson method;
12.2 Numerical integration	4	(e) use the trapezium rule; (f) use sketch graphs to determine whether the trapezium rule gives an over-estimate or an under-estimate in simple cases.

THIRD TERM: STATISTICS

<i>Topic</i>	<i>Teaching Period</i>	<i>Learning Outcome</i>
13 Data Description	14	<p>Candidates should be able to:</p> <ul style="list-style-type: none"> (a) identify discrete, continuous, ungrouped and grouped data; (b) construct and interpret stem-and-leaf diagrams, box-and-whisker plots, histograms and cumulative frequency curves; (c) state the mode and range of ungrouped data; (d) determine the median and interquartile range of ungrouped and grouped data; (e) calculate the mean and standard deviation of ungrouped and grouped data, from raw data and from given totals such as $\sum_{i=1}^n (x_i - a)$ and $\sum_{i=1}^n (x_i - a)^2$; (f) select and use the appropriate measures of central tendency and measures of dispersion; (g) calculate the Pearson coefficient of skewness; (h) describe the shape of a data distribution.
14 Probability	14	<p>Candidates should be able to:</p> <ul style="list-style-type: none"> (a) apply the addition principle and the multiplication principle; (b) use the formulae for combinations and permutations in simple cases; (c) identify a sample space, and calculate the probability of an event; (d) identify complementary, exhaustive and mutually exclusive events; (e) use the formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; (f) calculate conditional probabilities, and identify independent events; (g) use the formulae $P(A \cap B) = P(A) \times P(B A) = P(B) \times P(A B)$; (h) use the rule of total probability.

<i>Topic</i>	<i>Teaching Period</i>	<i>Learning Outcome</i>
15 Probability Distributions	26	Candidates should be able to:
15.1 Discrete random variables	6	(a) identify discrete random variables; (b) construct a probability distribution table for a discrete random variable; (c) use the probability function and cumulative distribution function of a discrete random variable; (d) calculate the mean and variance of a discrete random variable;
15.2 Continuous random variables	6	(e) identify continuous random variables; (f) relate the probability density function and cumulative distribution function of a continuous random variable; (g) use the probability density function and cumulative distribution function of a continuous random variable; (h) calculate the mean and variance of a continuous random variable;
15.3 Binomial distribution	4	(i) use the probability function of a binomial distribution, and find its mean and variance; (j) use the binomial distribution as a model for solving problems related to science and technology;
15.4 Poisson distribution	4	(k) use the probability function of a Poisson distribution, and identify its mean and variance; (l) use the Poisson distribution as a model for solving problems related to science and technology;
15.5 Normal distribution	6	(m) identify the general features of a normal distribution, in relation to its mean and standard deviation; (n) standardise a normal random variable and use the normal distribution tables; (o) use the normal distribution as a model for solving problems related to science and technology; (p) use the normal distribution, with continuity correction, as an approximation to the binomial distribution, where appropriate.

<i>Topic</i>	<i>Teaching Period</i>	<i>Learning Outcome</i>
16 Sampling and Estimation	26	Candidates should be able to:
16.1 Sampling	14	<ul style="list-style-type: none"> (a) distinguish between a population and a sample, and between a parameter and a statistic; (b) identify a random sample; (c) identify the sampling distribution of a statistic; (d) determine the mean and standard deviation of the sample mean; (e) use the result that \bar{X} has a normal distribution if X has a normal distribution; (f) use the central limit theorem; (g) determine the mean and standard deviation of the sample proportion; (h) use the approximate normality of the sample proportion for a sufficiently large sample size;
16.2 Estimation	12	<ul style="list-style-type: none"> (i) calculate unbiased estimates for the population mean and population variance; (j) calculate an unbiased estimate for the population proportion; (k) determine and interpret a confidence interval for the population mean based on a sample from a normally distributed population with known variance; (l) determine and interpret a confidence interval for the population mean based on a large sample; (m) find the sample size for the estimation of population mean; (n) determine and interpret a confidence interval for the population proportion based on a large sample; (o) find the sample size for the estimation of population proportion.

<i>Topic</i>	<i>Teaching Period</i>	<i>Learning Outcome</i>
17 Hypothesis Testing	14	<p>Candidates should be able to:</p> <ul style="list-style-type: none"> (a) explain the meaning of a null hypothesis and an alternative hypothesis; (b) explain the meaning of the significance level of a test; (c) carry out a hypothesis test concerning the population mean for a normally distributed population with known variance; (d) carry out a hypothesis test concerning the population mean in the case where a large sample is used; (e) carry out a hypothesis test concerning the population proportion by direct evaluation of binomial probabilities; (f) carry out a hypothesis test concerning the population proportion using a normal approximation.
18 Chi-squared Tests	14	<p>Candidates should be able to:</p> <ul style="list-style-type: none"> (a) identify the shape, as well as the mean and variance, of a chi-squared distribution with a given number of degrees of freedom; (b) use the chi-squared distribution tables; (c) identify the chi-squared statistic; (d) use the result that classes with small expected frequencies should be combined in a chi-squared test; (e) carry out goodness-of-fit tests to fit prescribed probabilities and probability distributions with known parameters; (f) carry out tests of independence in contingency tables (excluding Yates correction).

Coursework

The Mathematics (T) coursework is intended to enable candidates to carry out mathematical investigation and mathematical modelling, so as to enhance the understanding of mathematical processes and applications and to develop soft skills.

The coursework comprises three assignments set down by the Malaysian Examinations Council. The assignments are based on three different areas of the syllabus and represent two types of tasks: mathematical investigation and mathematical modelling.

A school candidate is required to carry out one assignment in each term under the supervision of the subject teacher as specified in the Teacher's Manual for Mathematics (T) Coursework which can be downloaded from MEC's Portal (<http://www.mpm.edu.my>) by the subject teacher during the first term of form six. The assignment reports are graded by the subject teacher in the respective terms. A viva session is conducted by the teacher in each term after the assessment of the assignment reports.

An individual private candidate is required to carry out one assignment in each term as specified in the Individual Private Candidate's Manual for Mathematics (T) Coursework which can be downloaded from MEC's Portal (<http://www.mpm.edu.my>) by the candidate during the first term of form six. The assignment reports are graded by an external examiner in the respective terms. A viva session is conducted by the examiner in each term after the assessment of the assignment reports.

A repeating candidate may use the total mark obtained in the coursework for the subsequent STPM examination. Requests to carry forward the moderated coursework mark should be made during the registration of the examination.

Scheme of Assessment

<i>Term of Study</i>	<i>Paper Code and Name</i>	<i>Type of Test</i>	<i>Mark (Weighting)</i>	<i>Duration</i>	<i>Administration</i>
First Term	954/1 Mathematics (T) Paper 1	Written test Section A Answer all 6 questions of variable marks. Section B Answer 1 out of 2 questions. All questions are based on topics 1 to 6.	60 (26.67%) 45 15	1½ hours	Central assessment
Second Term	954/2 Mathematics (T) Paper 2	Written test Section A Answer all 6 questions of variable marks. Section B Answer 1 out of 2 questions. All questions are based on topics 7 to 12.	60 (26.67%) 45 15	1½ hours	Central assessment
Third Term	954/3 Mathematics (T) Paper 3	Written test Section A Answer all 6 questions of variable marks. Section B Answer 1 out of 2 questions. All questions are based on topics 13 to 18.	60 (26.67%) 45 15	1½ hours	Central assessment
First, Second and Third Terms	954/4 Mathematics (T) Paper 4	Coursework 3 assignments, each based on topics 1 to 6, topics 7 to 12 and topics 13 to 18.	180 to be scaled to 45 (20%)	Throughout the three terms	Assessment by school teachers for candidates from government and government-aided schools Assessment by appointed assessors for candidates from private schools and individual private candidates

Performance Descriptions

A grade **A** candidate is likely able to:

- (a) use correctly mathematical concepts, terminology and notation;
- (b) display and interpret mathematical information in tabular, diagrammatic and graphical forms;
- (c) identify mathematical patterns and structures in a variety of situations;
- (d) use appropriate mathematical models in different contexts;
- (e) apply correctly mathematical principles and techniques in solving problems;
- (f) carry out calculations and approximations to an appropriate degree of accuracy;
- (g) interpret the significance and reasonableness of results, making sensible predictions where appropriate;
- (h) present mathematical explanations, arguments and conclusions, usually in a logical and systematic manner.

A grade **C** candidate is likely able to:

- (a) use correctly some mathematical concepts, terminology and notation;
- (b) display and interpret some mathematical information in tabular, diagrammatic and graphical forms;
- (c) identify mathematical patterns and structures in certain situations;
- (d) use appropriate mathematical models in certain contexts;
- (e) apply correctly some mathematical principles and techniques in solving problems;
- (f) carry out some calculations and approximations to an appropriate degree of accuracy;
- (g) interpret the significance and reasonableness of some results;
- (h) present some mathematical explanations, arguments and conclusions.

Mathematical Notation

Miscellaneous symbols

$=$	is equal to
\neq	is not equal to
\equiv	is identical to or is congruent to
\approx	is approximately equal to
$<$	is less than
\leq	is less than or equal to
$>$	is greater than
\geq	is greater than or equal to
∞	infinity
\therefore	therefore

Operations

$a + b$	a plus b
$a - b$	a minus b
$a \times b, ab$	a multiplied by b
$a \div b, \frac{a}{b}$	a divided by b
$a : b$	ratio of a to b
a^n	n th power of a
$a^{\frac{1}{2}}, \sqrt{a}$	positive square root of a
$a^{\frac{1}{n}}, \sqrt[n]{a}$	positive n th root of a
$ a $	absolute value of a real number a
$\sum_{i=1}^n u_i$	$u_1 + u_2 + \dots + u_n$
$n!$	n factorial for $n \in \mathbb{N}$
$\binom{n}{r}$	binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n, r \in \mathbb{N}, 0 < r < n$

Set notation

\in	is an element of
\notin	is not an element of
\emptyset	empty set
$\{x \mid \dots\}$	set of x such that \dots
\mathbb{N}	set of natural numbers, $\{0, 1, 2, 3, \dots\}$
\mathbb{Z}	set of integers
\mathbb{Z}^+	set of positive integers
\mathbb{Q}	set of rational numbers
\mathbb{R}	set of real numbers

$[a, b]$	closed interval $\{x \mid x \in \mathbb{R}, a \leq x < b\}$
(a, b)	open interval $\{x \mid x \in \mathbb{R}, a < x < b\}$
$[a, b)$	interval $\{x \mid x \in \mathbb{R}, a \leq x < b\}$
$(a, b]$	interval $\{x \mid x \in \mathbb{R}, a < x \leq b\}$
\cup	union
\cap	intersection

Functions

f	a function f
$f(x)$	value of a function f at x
$f : A \rightarrow B$	f is a function under which each element of set A has an image in set B
$f : x \mapsto y$	f is a function which maps the element x to the element y
f^{-1}	inverse function of f
$f \circ g$	composite function of f and g which is defined by $f \circ g(x) = f[g(x)]$
e^x	exponential function of x
$\log_a x$	logarithm to base a of x
$\ln x$	natural logarithm of x , $\log_e x$
$\left. \begin{array}{l} \sin, \cos, \tan, \\ \csc, \sec, \cot \end{array} \right\}$	trigonometric functions
$\left. \begin{array}{l} \sin^{-1}, \cos^{-1}, \tan^{-1}, \\ \csc^{-1}, \sec^{-1}, \cot^{-1} \end{array} \right\}$	inverse trigonometric functions

Matrices

\mathbf{A}	a matrix \mathbf{A}
$\mathbf{0}$	null matrix
\mathbf{I}	identity matrix
\mathbf{A}^T	transpose of a matrix \mathbf{A}
\mathbf{A}^{-1}	inverse of a non-singular square matrix \mathbf{A}
$\det \mathbf{A}$	determinant of a square matrix \mathbf{A}

Complex numbers

i	square root of -1
z	a complex number z
$ z $	modulus of z
$\arg z$	argument of z
z^*	complex conjugate of z

Geometry

AB	length of the line segment with end points A and B
$\angle BAC$	angle between the line segments AB and AC
$\triangle ABC$	triangle whose vertices are A , B and C
$//$	is parallel to
\perp	is perpendicular to

Vectors

\mathbf{a}	a vector \mathbf{a}
$ \mathbf{a} $	magnitude of a vector \mathbf{a}
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinates axes
\overline{AB}	vector represented in magnitude and direction by the directed line segment from point A to point B
$ \overline{AB} $	magnitude of \overline{AB}
$\mathbf{a} \cdot \mathbf{b}$	scalar product of vectors \mathbf{a} and \mathbf{b}
$\mathbf{a} \times \mathbf{b}$	vector product of vectors \mathbf{a} and \mathbf{b}

Derivatives and integrals

$\lim_{x \rightarrow a} f(x)$	limit of $f(x)$ as x tends to a
$\frac{dy}{dx}$	first derivative of y with respect to x
$f'(x)$	first derivative of $f(x)$ with respect to x
$\frac{d^2 y}{dx^2}$	second derivative of y with respect to x
$f''(x)$	second derivative of $f(x)$ with respect to x
$\frac{d^n y}{dx^n}$	n th derivative of y with respect to x
$f^{(n)}(x)$	n th derivative of $f(x)$ with respect to x
$\int y \, dx$	indefinite integral of y with respect to x
$\int_a^b y \, dx$	definite integral of y with respect to x for values of x between a and b

Data description

x_1, x_2, \dots	observations
f_1, f_2, \dots	frequencies with which the observations x_1, x_2, \dots occur
\bar{x}	sample mean
s^2	sample variance, $s^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$
μ	population mean
σ^2	population variance

Probability

A	an event A
A'	complement of an event A or the event not A
$P(A)$	probability of an event A
$P(A B)$	probability of event A given event B

Probability distributions

X	a random variable X
x	value of a random variable X
Z	standardised normal random variable
z	value of the standardised normal random variable Z
$f(x)$	value of the probability density function of a continuous random variable X
$F(x)$	value of the cumulative distribution function of a continuous random variable X
$E(X)$	expectation of a random variable X
$\text{Var}(X)$	variance of a random variable X
$B(n, p)$	binomial distribution with parameters n and p
$\text{Po}(\lambda)$	Poisson distribution with parameter λ
$N(\mu, \sigma^2)$	normal distribution with mean μ and variance σ^2
χ^2_ν	chi-squared distribution with ν degrees of freedom

Sampling and estimation

$\hat{\mu}$	unbiased estimate of the population mean
$\hat{\sigma}^2$	unbiased estimate of the population variance
p	population proportion
\hat{p}	sample proportion

Electronic Calculators

During the written paper examination, candidates are advised to have standard scientific calculators which must be silent. Programmable and graphic display calculators are prohibited.

Reference Books

Teachers and candidates may use books specially written for the STPM examination and other reference books such as those listed below.

Algebra and Geometry

1. Harcet, J., Heinrichs, L., Seiler, P.M. and Skoumal, M.T., 2012. *Mathematics: Higher Level, IB Diploma Programme*. United Kingdom: Oxford University Press.
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SPECIMEN PAPER

954/1

STPM

MATHEMATICS (T) (MATEMATIK (T))

PAPER 1 (KERTAS 1)

One and a half hours (Satu jam setengah)

MAJLIS PEPERIKSAAN MALAYSIA
(MALAYSIAN EXAMINATIONS COUNCIL)

SIJIL TINGGI PERSEKOLAHAN MALAYSIA
(MALAYSIA HIGHER SCHOOL CERTIFICATE)

Instruction to candidates:

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

*Answer **all** questions in Section A and any **one** question in Section B. Answers may be written in either English or Bahasa Malaysia.*

All necessary working should be shown clearly.

Scientific calculators may be used. Programmable and graphic display calculators are prohibited.

A list of mathematical formulae is provided on page of this question paper.

Arahan kepada calon:

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU UNTUK BERBUAT DEMIKIAN.

*Jawab **semua** soalan dalam Bahagian A dan mana-mana **satu** soalan dalam Bahagian B. Jawapan boleh ditulis dalam bahasa Inggeris atau Bahasa Malaysia.*

Semua kerja yang perlu hendaklah ditunjukkan dengan jelas.

Kalkulator saintifik boleh digunakan. Kalkulator boleh atur cara dan kalkulator paparan grafik tidak dibenarkan.

Senarai rumus matematik dibekalkan pada halaman dalam kertas soalan ini.

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Section A [45 marks]

Answer **all** questions in this section.

- 1 The functions f and g are defined by

$$f : x \mapsto e^{2x}, x \in \mathbb{R};$$

$$g : x \mapsto (\ln x)^2, x > 0.$$

- (a) Find f^{-1} and state its domain. [3 marks]

- (b) Show that $g\left(\frac{1}{2}\right) = g(2)$, and state, with a reason, whether g has an inverse. [4 marks]

- 2 A sequence is defined by $u_r = e^{-(r-1)} - e^{-r}$ for all integers $r > 1$. Find $\sum_{r=1}^n u_r$, in terms of n , and

deduce the value of $\sum_{r=1}^{\infty} u_r$. [5 marks]

- 3 The matrices $\mathbf{P} = \begin{pmatrix} 2 & -2 & 0 \\ 0 & 0 & 2 \\ a & b & c \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} -1 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & -2 & 2 \end{pmatrix}$ are such that $\mathbf{PQ} = \mathbf{QP}$.

- (a) Determine the values of a , b and c . [5 marks]

- (b) Find the real numbers m and n for which $\mathbf{P} = m\mathbf{Q} + n\mathbf{I}$, where \mathbf{I} is the 3×3 identity matrix. [5 marks]

- 4 Express the complex number $z = 1 - \sqrt{3}i$ in polar form. [4 marks]

Hence, find $z^5 + \frac{1}{z^5}$ and $z^5 - \frac{1}{z^5}$. [4 marks]

- 5 The equation of a hyperbola is $4x^2 - 9y^2 - 24x - 18y - 9 = 0$.

- (a) Obtain the standard form for the equation of the hyperbola. [3 marks]

- (b) Find the vertices and the equations of the asymptotes of the hyperbola. [6 marks]

- 6 Find the equation of the plane which contains the straight line $x - 3 = \frac{y - 4}{3} = \frac{z + 1}{2}$ and is perpendicular to the plane $3x + 2y - z = 3$. [6 marks]

Bahagian A [45 markah]

Jawab semua soalan dalam bahagian ini.

1 Fungsi f dan g ditakrifkan oleh

$$f : x \mapsto e^{2x}, x \in \mathbb{R};$$

$$g : x \mapsto (\ln x)^2, x > 0.$$

(a) Cari f^{-1} dan nyatakan domainnya. [3 markah]

(b) Tunjukkan bahawa $g\left(\frac{1}{2}\right) = g(2)$, dan nyatakan, dengan satu sebab, sama ada g mempunyai songsangan. [4 markah]

2 Satu jujukan ditakrifkan oleh $u_r = e^{-(r-1)} - e^{-r}$ bagi semua integer $r > 1$. Cari $\sum_{r=1}^n u_r$, dalam sebutan n , dan deduksikan nilai $\sum_{r=1}^{\infty} u_r$. [5 markah]

3 Matriks $\mathbf{P} = \begin{pmatrix} 2 & -2 & 0 \\ 0 & 0 & 2 \\ a & b & c \end{pmatrix}$ dan $\mathbf{Q} = \begin{pmatrix} -1 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & -2 & 2 \end{pmatrix}$ adalah sebegitu rupa sehinggakan $\mathbf{PQ} = \mathbf{QP}$.

(a) Tentukan nilai a , b , dan c . [5 markah]

(b) Cari nombor nyata m dan n supaya $\mathbf{P} = m\mathbf{Q} + n\mathbf{I}$, dengan \mathbf{I} matriks identiti 3×3 . [5 markah]

4 Ungkapkan nombor kompleks $z = 1 - \sqrt{3}i$ dalam bentuk kutub. [4 markah]

Dengan yang demikian, cari $z^5 + \frac{1}{z^5}$ dan $z^5 - \frac{1}{z^5}$. [4 markah]

5 Persamaan satu hiperbola ialah $4x^2 - 9y^2 - 24x - 18y - 9 = 0$.

(a) Dapatkan bentuk piawai bagi persamaan hiperbola itu. [3 markah]

(b) Cari bucu dan persamaan asimptot hiperbola itu. [6 markah]

6 Cari persamaan satah yang mengandungi garis lurus $x - 3 = \frac{y - 4}{3} = \frac{z + 1}{2}$ dan serenjang dengan satah $3x + 2y - z = 3$. [6 markah]

Section B [15 marks]

Answer any **one** question in this section.

7 Express $\cos x + \sin x$ in the form $r \cos(x - \alpha)$, where $r > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Hence, find the minimum and maximum values of $\cos x + \sin x$ and the corresponding values of x in the interval $0 < x < 2\pi$. [7 marks]

(a) Sketch the graph of $y = \cos x + \sin x$ for $0 < x < 2\pi$. [3 marks]

(b) By drawing appropriate lines on your graph, determine the number of roots in the interval $0 < x < 2\pi$ of each of the following equations.

(i) $\cos x + \sin x = -\frac{1}{2}$ [1 mark]

(ii) $\cos x + \sin x = 2$ [1 mark]

(c) Find the set of values of x in the interval $0 < x < 2\pi$ for which $|\cos x + \sin x| > 1$. [3 marks]

8 The position vectors **a**, **b** and **c** of three points *A*, *B* and *C* respectively are given by

$$\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k},$$

$$\mathbf{b} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k},$$

$$\mathbf{c} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}.$$

(a) Find a unit vector parallel to $\mathbf{a} + \mathbf{b} + \mathbf{c}$. [3 marks]

(b) Calculate the acute angle between \mathbf{a} and $\mathbf{a} + \mathbf{b} + \mathbf{c}$. [3 marks]

(c) Find the vector of the form $\lambda\mathbf{j} + \mu\mathbf{k}$ perpendicular to both \mathbf{a} and \mathbf{b} . [2 marks]

(d) Determine the position vector of the point *D* which is such that *ABCD* is a parallelogram having *BD* as a diagonal. [3 marks]

(e) Calculate the area of the parallelogram *ABCD*. [4 marks]

Bahagian B [15 markah]

Jawab mana-mana **satu** soalan dalam bahagian ini.

7 Ungkapkan $\cos x + \sin x$ dalam bentuk $r \cos(x - \alpha)$, dengan $r > 0$ dan $0 < \alpha < \frac{1}{2}\pi$. Dengan yang demikian, cari nilai minimum dan nilai maksimum $\cos x + \sin x$ dan nilai x yang sepadan dalam selang $0 < x < 2\pi$. [7 markah]

(a) Lakar graf $y = \cos x + \sin x$ bagi $0 < x < 2\pi$. [3 markah]

(b) Dengan melukis garis yang sesuai pada graf anda, tentukan bilangan punca dalam selang $0 < x < 2\pi$ setiap persamaan yang berikut.

(i) $\cos x + \sin x = -\frac{1}{2}$ [1 markah]

(ii) $\cos x + \sin x = 2$ [1 markah]

(c) Cari set nilai x dalam selang $0 < x < 2\pi$ supaya $|\cos x + \sin x| > 1$. [3 markah]

8 Vektor kedudukan **a**, **b**, dan **c** tiga titik *A*, *B*, dan *C* masing-masing diberikan oleh

$$\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k},$$

$$\mathbf{b} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k},$$

$$\mathbf{c} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}.$$

(a) Cari vektor unit yang selari dengan $\mathbf{a} + \mathbf{b} + \mathbf{c}$. [3 markah]

(b) Hitung sudut tirus di antara **a** dengan $\mathbf{a} + \mathbf{b} + \mathbf{c}$. [3 markah]

(c) Cari vektor dalam bentuk $\lambda\mathbf{i} + \mu\mathbf{j} + \nu\mathbf{k}$ yang seranjang dengan kedua-dua **a** dan **b**. [2 markah]

(d) Tentukan vektor kedudukan titik *D* yang sebegitu rupa sehinggakan *ABCD* ialah satu segiempat selari dengan *BD* sebagai satu pepenjuru. [3 markah]

(e) Hitung luas segiempat selari *ABCD*. [4 markah]

MATHEMATICAL FORMULAE (RUMUS MATEMATIK)

Binomial expansions (Kembangan binomial)

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n, \quad n \in \mathbb{Z}^+$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \dots + \frac{n(n-1) \dots (n-r+1)}{r!} x^r + \dots, \quad n \in \mathbb{Q}, |x| < 1$$

Conics (Keratan kon)

Parabola with vertex (h, k) , focus $(a + h, k)$ and directrix $x = -a + h$
(Parabola dengan bucu (h, k) , fokus $(a + h, k)$ dan direktriks $x = -a + h$)

$$(y - k)^2 = 4a(x - h)$$

Ellipse with centre (h, k) and foci $(-c + h, k)$, $(c + h, k)$, where $c^2 = a^2 - b^2$
(Elips dengan pusat (h, k) dan fokus $(-c + h, k)$, $(c + h, k)$, dengan $c^2 = a^2 - b^2$)

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

Hyperbola with centre (h, k) and foci $(-c + h, k)$, $(c + h, k)$, where $c^2 = a^2 + b^2$
(Hiperbola dengan pusat (h, k) dan fokus $(-c + h, k)$, $(c + h, k)$, dengan $c^2 = a^2 + b^2$)

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

SPECIMEN PAPER

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STPM

MATHEMATICS (T) (MATEMATIK (T))

PAPER 2 (KERTAS 2)

One and a half hours (Satu jam setengah)

MAJLIS PEPERIKSAAN MALAYSIA
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STPM 954/2

Section A [45 marks]

Answer **all** questions in this section.

1 The function f is defined by

$$f(x) = \begin{cases} \sqrt{x+1}, & x \geq -1; \\ |x|-1, & \text{otherwise.} \end{cases}$$

(a) Find $\lim_{x \rightarrow -1} f(x)$. [3 marks]

(b) Determine whether f is continuous at $x = -1$. [2 marks]

2 Find the equation of the normal to the curve with parametric equations $x = 1 - 2t$ and $y = -2 + \frac{2}{t}$ at the point $(3, -4)$. [6 marks]

3 Using the substitution $x = 4\sin^2 u$, evaluate $\int_0^1 \sqrt{\frac{x}{4-x}} dx$. [6 marks]

4 Show that $e^{\int \frac{x-2}{x(x-1)} dx} = \frac{x^2}{x-1}$. [4 marks]

Hence, find the particular solution of the differential equation

$$\frac{dy}{dx} + \frac{x-2}{x(x-1)} y = -\frac{1}{x^2(x-1)}$$

which satisfies the boundary condition $y = \frac{3}{4}$ when $x = 2$. [4 marks]

5 If $y = \sin^{-1} x$, show that $\frac{d^2 y}{dx^2} = x \left(\frac{dy}{dx} \right)^3$ and $\frac{d^3 y}{dx^3} = \left(\frac{dy}{dx} \right)^3 + 3x^2 \left(\frac{dy}{dx} \right)^5$. [5 marks]

Using Maclaurin's theorem, express $\sin^{-1} x$ as a series of ascending powers of x up to the term in x^5 . State the range of values of x for which the expansion is valid. [7 marks]

6 Use the trapezium rule with subdivisions at $x = 3$ and $x = 5$ to obtain an approximation to $\int_1^7 \frac{x^3}{1+x^4} dx$, giving your answer correct to three places of decimals. [4 marks]

By evaluating the integral exactly, show that the error of the approximation is about 4.1%. [4 marks]

Bahagian A [45 markah]

Jawab semua soalan dalam bahagian ini.

1 Fungsi f ditakrifkan oleh

$$f(x) = \begin{cases} \sqrt{x+1}, & x \geq -1; \\ |x|-1, & \text{jika tidak.} \end{cases}$$

(a) Cari had $f(x)$ $\lim_{x \rightarrow -1}$. [3 markah]

(b) Tentukan sama ada f adalah selanjar di $x = -1$. [2 markah]

2 Cari persamaan normal kepada lengkung dengan persamaan berparameter $x = 1 - 2t$ dan $y = -2 + \frac{2}{t}$ di titik $(3, -4)$. [6 markah]

3 Dengan menggunakan gantian $x = 4\sin^2 u$, nilaikan $\int_0^1 \sqrt{\frac{x}{4-x}} dx$. [6 markah]

4 Tunjukkan bahawa $e^{\int \frac{x-2}{x(x-1)} dx} = \frac{x^2}{x-1}$. [4 markah]

Dengan yang demikian, cari penyelesaian khusus persamaan pembezaan

$$\frac{dy}{dx} + \frac{x-2}{x(x-1)} y = -\frac{1}{x^2(x-1)}$$

yang memenuhi syarat sempadan $y = \frac{3}{4}$ apabila $x = 2$. [4 markah]

5 Jika $y = \sin^{-1} x$, tunjukkan bahawa $\frac{d^2 y}{dx^2} = x \left(\frac{dy}{dx} \right)^3$ dan $\frac{d^3 y}{dx^3} = \left(\frac{dy}{dx} \right)^3 + 3x^2 \left(\frac{dy}{dx} \right)^5$. [5 markah]

Dengan menggunakan teorem Maclaurin, ungkapkan $\sin^{-1} x$ sebagai satu siri kuasa x menaik hingga sebutan dalam x^5 . Nyatakan julat nilai x supaya kembangan itu sah. [7 markah]

6 Gunakan petua trapezium dengan subbahagian di $x = 3$ dan $x = 5$ untuk memperoleh penghampiran $\int_1^7 \frac{x^3}{1+x^4} dx$, dengan memberikan jawapan anda betul hingga tiga tempat perpuluhan. [4 markah]

Dengan menilai kamiran itu secara tepat, tunjukkan bahawa ralat penghampiran adalah lebih kurang 4.1%. [4 markah]

Section B [15 marks]

Answer any **one** question in this section.

7 A right circular cone of height $a + x$, where $-a < x < a$, is inscribed in a sphere of constant radius a , such that the vertex and all points on the circumference of the base lie on the surface of the sphere.

(a) Show that the volume V of the cone is given by $V = \frac{1}{3}\pi(a - x)(a + x)^2$. [3 marks]

(b) Determine the value of x for which V is maximum and find the maximum value of V . [6 marks]

(c) Sketch the graph of V against x . [2 marks]

(d) Determine the rate at which V changes when $x = \frac{1}{2}a$ if x is increasing at a rate of $\frac{1}{10}a$ per minute. [4 marks]

8 Two iterations suggested to estimate a root of the equation $x^3 - 4x^2 + 6 = 0$ are $x_{n+1} = 4 - \frac{6}{x_n^2}$ and $x_{n+1} = \frac{1}{2}(x_n^3 + 6)^{\frac{1}{2}}$.

(a) Show that the equation $x^3 - 4x^2 + 6 = 0$ has a root between 3 and 4. [3 marks]

(b) Using sketched graphs of $y = x$ and $y = f(x)$ on the same axes, show that, with initial approximation $x_0 = 3$, one of the iterations converges to the root whereas the other does not. [6 marks]

(c) Use the iteration which converges to the root to obtain a sequence of iterations with $x_0 = 3$, ending the process when the difference of two consecutive iterations is less than 0.05. [4 marks]

(d) Determine whether the iteration used still converges to the root if the initial approximation is $x_0 = 4$. [2 marks]

Bahagian B [15 markah]

Jawab mana-mana **satu** soalan dalam bahagian ini.

7 Satu kon bulat tegak dengan tinggi $a + x$, dengan $-a < x < a$, diterapkan di dalam satu sfera berjejari malar a , sebegitu rupa sehinggakan bucu dan semua titik pada lilitan tapak terletak pada permukaan sfera itu.

(a) Tunjukkan bahawa isi padu V kon itu diberikan oleh $V = \frac{1}{3}\pi(a-x)(a+x)^2$. [3 markah]

(b) Tentukan nilai x supaya V maksimum dan cari nilai maksimum V . [6 markah]

(c) Lakar graf V lawan x . [2 markah]

(d) Tentukan kadar V berubah apabila $x = \frac{1}{2}a$ jika x menokok pada kadar $\frac{1}{10}a$ per minut. [4 markah]

8 Dua lelaran yang dicadangkan untuk menganggar satu punca persamaan $x^3 - 4x^2 + 6 = 0$ ialah $x_{n+1} = 4 - \frac{6}{x_n^2}$ dan $x_{n+1} = \frac{1}{2}(x_n^3 + 6)^{\frac{1}{2}}$.

(a) Tunjukkan bahawa persamaan $x^3 - 4x^2 + 6 = 0$ mempunyai satu punca antara 3 dengan 4. [3 markah]

(b) Dengan menggunakan lakaran graf $y = x$ dan $y = f(x)$ pada paksi yang sama, tunjukkan bahawa, dengan penghampiran awal $x_0 = 3$, salah satu lelaran menumpu kepada punca itu sedangkan yang lain tidak. [6 markah]

(c) Gunakan lelaran yang menumpu kepada punca itu untuk memperoleh satu jujukan lelaran dengan $x_0 = 3$, dengan menamatkan proses apabila beza dua lelaran yang berturut-turut kurang daripada 0.05. [4 markah]

(d) Tentukan sama ada lelaran yang digunakan masih menumpu kepada punca itu jika penghampiran awal ialah $x_0 = 4$. [2 markah]

MATHEMATICAL FORMULAE (RUMUS MATEMATIK)

Differentiation (Pembezaan)

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

Integration (Pengamiran)

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Maclaurin series (Siri Maclaurin)

$$e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^r}{r!} + \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{r+1} \frac{x^r}{r} + \dots, \quad -1 < x \leq 1$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^r \frac{x^{2r}}{(2r)!} + \dots$$

Numerical methods (Kaedah berangka)

Newton-Raphson method (Kaedah Newton-Raphson)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, 3, \dots$$

Trapezium rule (Petua trapezium)

$$\int_a^b y dx \approx \frac{1}{2}h[y_0 + 2(y_1 + y_2 + \dots + y_{n-1}) + y_n], \quad h = \frac{b-a}{n}$$

SPECIMEN PAPER

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STPM

MATHEMATICS (T) (MATEMATIK (T))

PAPER 3 (KERTAS 3)

One and a half hours (Satu jam setengah)

MAJLIS PEPERIKSAAN MALAYSIA
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All necessary working should be shown clearly.

Scientific calculators may be used. Programmable and graphic display calculators are prohibited.

A list of mathematical formulae, statistical tables and a graph paper are provided on pages to of this question paper.

Arahan kepada calon:

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Senarai rumus matematik, sifir statistik, dan satu kertas graf dibekalkan pada halaman hingga dalam kertas soalan ini.

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Section A [45 marks]

Answer **all** questions in this section.

1 The number of ships anchored at a port is recorded every week. The results for 26 particular weeks are as follows:

32 28 43 21 35 19 25 45 35 32 18 26 30
26 27 38 42 18 37 50 46 23 40 20 29 46

- (a) Display the data in a stem-and-leaf diagram. [2 marks]
- (b) Find the median and interquartile range. [4 marks]
- (c) Draw a box-and-whisker plot to represent the data. [3 marks]
- (d) State the shape of the frequency distribution, giving a reason for your answer. [2 marks]
- 2** The events A and B are such that $P(A) \neq 0$ and $P(B) \neq 0$.
- (a) Show that $P(A' | B) = 1 - P(A | B)$. [2 marks]
- (b) Show that $P(A' | B) = P(A')$ if A and B are independent. [3 marks]
- 3** The number of defective electrical components per 1000 components manufactured on a machine may be modelled by a Poisson distribution with a mean of 4.
- (a) Calculate the probability that there are at most 3 defective electrical components in the next 100 components manufactured on the machine. [3 marks]
- (b) State the assumptions that need to be made about the defective electrical components in order that the Poisson distribution is a suitable model. [2 marks]
- 4** The masses of bags of flour produced in a factory have mean 1.004 kg and standard deviation 0.006 kg.
- (a) Find the probability that a randomly selected bag has a mass of at least 1 kg. State any assumptions made. [4 marks]
- (b) Find the probability that the mean mass of 50 randomly selected bags is at least 1 kg. [4 marks]
- 5** The proportion of fans of a certain football club who are able to explain the offside rule correctly is p . A random sample of 9 fans of the football club is selected and 6 fans are able to explain the offside rule correctly. Test the null hypothesis $H_0: p = 0.8$ against the alternative hypothesis $H_1: p < 0.8$ at the 10% significance level. [6 marks]

Bahagian A [45 markah]

Jawab semua soalan dalam bahagian ini.

1 Bilangan kapal yang berlabuh di pelabuhan direkodkan setiap minggu. Keputusan selama 26 minggu adalah seperti yang berikut:

32	28	43	21	35	19	25	45	35	32	18	26	30
26	27	38	42	18	37	50	46	23	40	20	29	46

- (a) Paparkan data itu dalam satu gambar rajah tangkai dan daun. [2 markah]
(b) Cari median dan julat antara kuartil. [4 markah]
(c) Lukis satu plot kotak dan misai untuk mewakili data itu. [3 markah]
(d) Nyatakan bentuk taburan kekerapan, dengan memberikan satu sebab bagi jawapan anda. [2 markah]

2 Peristiwa A dan B adalah sebegitu rupa sehinggakan $P(A) \neq 0$ dan $P(B) \neq 0$.

- (a) Tunjukkan bahawa $P(A' | B) = 1 - P(A | B)$. [2 markah]
(b) Tunjukkan bahawa $P(A' | B) = P(A')$ jika A dan B adalah tak bersandar. [3 markah]

3 Bilangan komponen elektrik yang cacat per 1000 komponen yang dikeluarkan pada satu mesin boleh dimodelkan oleh satu taburan Poisson dengan min 4.

- (a) Hitung kebarangkalian bahawa terdapat selebih-lebihnya 3 komponen elektrik yang cacat dalam 100 komponen yang berikutnya yang dikeluarkan pada mesin itu. [3 markah]
(b) Nyatakan anggapan yang perlu dibuat tentang komponen elektrik yang cacat itu supaya taburan Poisson ialah model yang sesuai. [2 markah]

4 Jisim beg tepung yang dihasilkan di sebuah kilang mempunyai min 1.004 kg dan sisihan piawai 0.006 kg.

- (a) Cari kebarangkalian bahawa sebuah beg yang dipilih secara rawak mempunyai jisim sekurang-kurangnya 1 kg. Nyatakan sebarang andaian yang dibuat. [4 markah]
(b) Cari kebarangkalian bahawa min jisim 50 beg yang dipilih secara rawak sekurang-kurangnya 1 kg. [4 markah]

5 Perkadaran peminat kelab bola sepak tertentu yang mampu menjelaskan peraturan ofsaid dengan betul ialah p . Satu sampel rawak 9 peminat bola sepak itu dipilih dan 6 peminat mampu menjelaskan peraturan ofsaid dengan betul. Uji hipotesis nol $H_0: p = 0.8$ terhadap hipotesis alternatif $H_1: p < 0.8$ pada aras keertian 10%. [6 markah]

6 It is thought that there is an association between the colour of a person's eyes and the reaction of the person's skin to ultraviolet light. In order to investigate this, each of a random sample of 120 persons is subjected to a standard dose of ultraviolet light. The degree of the reaction for each person is noted, “-” indicating no reaction, “+” indicating a slight reaction and “++” indicating a strong reaction. The results are shown in the table below.

<i>Reaction</i> \ <i>Eye colour</i>	<i>Blue</i>	<i>Grey or green</i>	<i>Brown</i>
-	7	8	18
+	29	10	16
++	21	9	2

Test whether the data provide evidence, at the 5% significance level, that the colour of a person's eyes and the reaction of the person's skin to ultraviolet light are independent. [10 marks]

6 Dipercayai bahawa terdapat perkaitan antara warna mata seseorang dengan tindak balas kulit orang itu terhadap cahaya ultra ungu. Untuk menyiasat perkara ini, setiap orang daripada satu sampel rawak 120 orang diberi dos piawai cahaya ultra ungu. Darjah tindak balas bagi setiap orang dicatat, dengan “-” menandakan tiada tindak balas, “+” menandakan sedikit tindak balas dan “++” menandakan tindak balas yang kuat. Keputusan ditunjukkan di dalam jadual di bawah.

<i>Warna mata</i> <i>Tindak balas</i>	<i>Biru</i>	<i>Kelabu atau hijau</i>	<i>Coklat</i>
-	7	8	18
+	29	10	16
++	21	9	2

Uji sama ada data itu memberikan bukti, pada aras keertian 5%, bahawa warna mata seseorang dan tindak balas kulit orang itu terhadap cahaya ultra ungu adalah tak bersandar. [10 markah]

Section B [15 marks]

Answer any **one** question in this section.

7 A random variable T , in hours, represents the life-span of a thermal detection system. The probability that the system fails to work at time t hour is given by

$$P(T < t) = 1 - e^{-\frac{21}{5500}t}.$$

- (a) Find the probability that the system works continuously for at least 250 hours. [3 marks]
- (b) Calculate the median life-span of the system. [3 marks]
- (c) Find the probability density function of the life-span of the system and sketch its graph. [4 marks]
- (d) Calculate the expected life-span of the system. [5 marks]

8 A random sample of 48 mushrooms is taken from a farm. The diameter x , in centimetres, of each mushroom is measured. The results are summarised by $\sum_{i=1}^{48} x_i = 300.4$ and $\sum_{i=1}^{48} x_i^2 = 2011.01$.

- (a) Calculate unbiased estimates of the population mean and variance of the diameters of the mushrooms. [3 marks]
- (b) Determine a 90% confidence interval for the mean diameter of the mushrooms. [4 marks]
- (c) Test, at the 10% significance level, the null hypothesis that the mean diameter of the mushrooms is 6.5cm. [6 marks]
- (d) Relate the confidence interval obtained in (b) with the result of the test in (c). [2 marks]

Bahagian B [15 markah]

Jawab mana-mana **satu** soalan dalam bahagian ini.

7 Pembolehubah rawak T , dalam jam, mewakili jangka hayat satu sistem pengesan haba. Kebarangkalian bahawa sistem itu gagal berfungsi pada masa t jam diberikan oleh

$$P(T < t) = 1 - e^{-\frac{21}{5500}t}.$$

(a) Cari kebarangkalian bahawa sistem itu berfungsi secara berterusan sekurang-kurangnya 250 jam. [3 markah]

(b) Hitung median jangka hayat sistem itu. [3 markah]

(c) Cari fungsi ketumpatan kebarangkalian jangka hayat sistem itu dan lakar grafnya. [4 markah]

(d) Hitung jangkaan jangka hayat sistem itu. [5 markah]

8 Satu sampel rawak 48 cendawan diambil dari sebuah ladang. Garis pusat x , dalam sentimeter, setiap cendawan diukur. Keputusan diiktisarkan oleh $\sum_{i=1}^{48} x_i = 300.4$ dan $\sum_{i=1}^{48} x_i^2 = 2011.01$.

(a) Hitung anggaran saksama min dan varians populasi garis pusat cendawan itu. [3 markah]

(b) Tentukan satu selang keyakinan 90% bagi min garis pusat cendawan itu. [4 markah]

(c) Uji, pada aras keertian 10%, hipotesis nol bahawa min garis pusat cendawan itu ialah 6.5 cm. [6 markah]

(d) Hubungkan selang keyakinan yang diperolehi dalam (b) dengan keputusan ujian dalam (c). [2 markah]

MATHEMATICAL FORMULAE (RUMUS MATEMATIK)

Probability distributions (Taburan kebarangkalian)

Binomial distribution (Taburan binomial)

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n$$

Poisson distribution (Taburan Poisson)

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad x = 0, 1, 2, \dots$$

Chi-squared tests (Ujian khi kuasa dua)

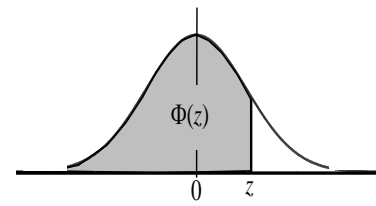
Test statistic (Statistik ujian)

$$\chi^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i}$$

THE NORMAL DISTRIBUTION FUNCTION (FUNGSI TABURAN NORMAL)

If Z has a normal distribution with mean 0 and variance 1, then for each value of z , the tabulated value of $\Phi(z)$ is such that $\Phi(z) = P(Z < z)$. For negative values of z , use $\Phi(-z) = 1 - \Phi(z)$.

Jika Z mempunyai taburan normal dengan min 0 dan varians 1, maka bagi setiap nilai z , nilai terjadual $\Phi(z)$ adalah sebegitu rupa sehinggakan $\Phi(z) = P(Z < z)$. Bagi nilai negatif z , gunakan $\Phi(-z) = 1 - \Phi(z)$.



z											ADD (TAMBAH)								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	31	35
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	8	11	15	19	23	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	21	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	6	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	6	9	11	14	17	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	11	14	16	18	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	19
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	5	7	9	11	13	15	16
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	3	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	2	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	1	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

Critical values for the normal distribution (Nilai genting bagi taburan normal)

If Z has a normal distribution with mean 0 and variance 1, then for each value of p , the tabulated value of z is such that $P(Z < z) = p$.

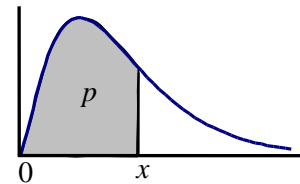
Jika Z mempunyai taburan normal dengan min 0 dan varians 1, maka bagi setiap nilai p , nilai terjadual z adalah sebegitu rupa sehinggakan $P(Z < z) = p$.

p	0.75	0.9	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

CRITICAL VALUES FOR THE χ^2 -DISTRIBUTION (NILAI GENTING BAGI TABURAN χ^2)

If X has a χ^2 -distribution with ν degrees of freedom, then for each pair of values of p and ν , the tabulated value of x is such that $P(X < x) = p$.

Jika X mempunyai taburan χ^2 dengan ν derajat kebebasan, maka bagi setiap pasangan nilai p dan ν , nilai terjadual x adalah sebegitu rupa sehinggakan $P(X < x) = p$.

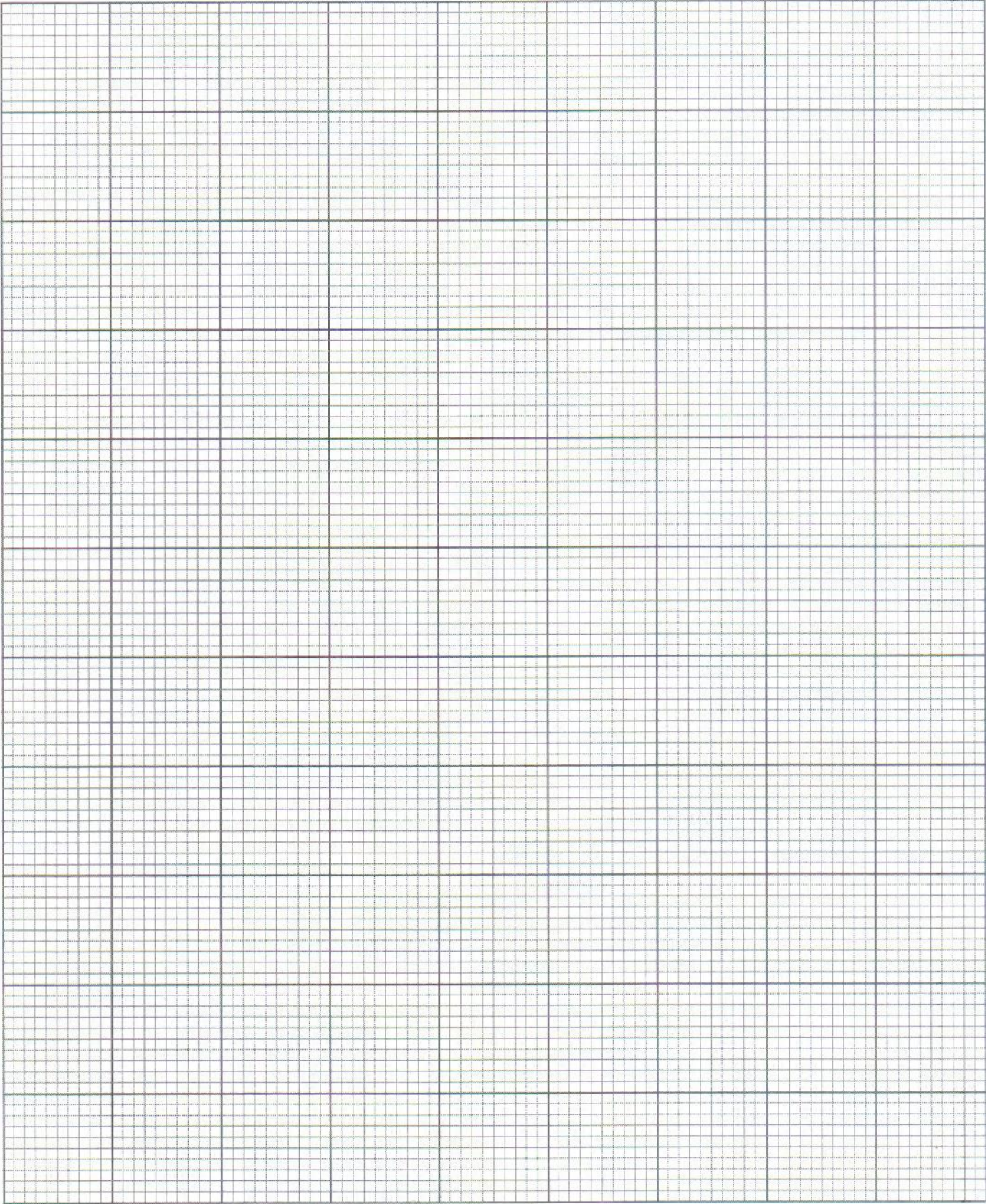


p	0.01	0.025	0.05	0.9	0.95	0.975	0.99	0.995	0.999
$\nu = 1$	0.01571	0.009821	0.023932	2.706	3.841	5.024	6.635	7.879	10.83
2	0.02010	0.05064	0.1026	4.605	5.991	7.378	9.210	10.60	13.82
3	0.1148	0.2158	0.3518	6.251	7.815	9.348	11.34	12.84	16.27
4	0.2971	0.4844	0.7107	7.779	9.488	11.14	13.28	14.86	18.47
5	0.5543	0.8312	1.145	9.236	11.07	12.83	15.09	16.75	20.51
6	0.8721	1.237	1.635	10.64	12.59	14.45	16.81	18.55	22.46
7	1.239	1.690	2.167	12.02	14.07	16.01	18.48	20.28	24.32
8	1.647	2.180	2.733	13.36	15.51	17.53	20.09	21.95	26.12
9	2.088	2.700	3.325	14.68	16.92	19.02	21.67	23.59	27.88
10	2.558	3.247	3.940	15.99	18.31	20.48	23.21	25.19	29.59
11	3.053	3.816	4.575	17.28	19.68	21.92	24.73	26.76	31.26
12	3.571	4.404	5.226	18.55	21.03	23.34	26.22	28.30	32.91
13	4.107	5.009	5.892	19.81	22.36	24.74	27.69	29.82	34.53
14	4.660	5.629	6.571	21.06	23.68	26.12	29.14	31.32	36.12
15	5.229	6.262	7.261	22.31	25.00	27.49	30.58	32.80	37.70
16	5.812	6.908	7.962	23.54	26.30	28.85	32.00	34.27	39.25
17	6.408	7.564	8.672	24.77	27.59	30.19	33.41	35.72	40.79
18	7.015	8.231	9.390	25.99	28.87	31.53	34.81	37.16	42.31
19	7.633	8.907	10.12	27.20	30.14	32.85	36.19	38.58	43.82
20	8.260	9.591	10.85	28.41	31.41	34.17	37.57	40.00	45.31
21	8.897	10.28	11.59	29.62	32.67	35.48	38.93	41.40	46.80
22	9.542	10.98	12.34	30.81	33.92	36.78	40.29	42.80	48.27
23	10.20	11.69	13.09	32.01	35.17	38.08	41.64	44.18	49.73
24	10.86	12.40	13.85	33.20	36.42	39.36	42.98	45.56	51.18
25	11.52	13.12	14.61	34.38	37.65	40.65	44.31	46.93	52.62
26	12.20	13.84	15.38	35.56	38.89	41.92	45.64	48.29	54.05
27	12.88	14.57	16.15	36.74	40.11	43.19	46.96	49.65	55.48
28	13.56	15.31	16.93	37.92	41.34	44.46	48.28	50.99	56.89
29	14.26	16.05	17.71	39.09	42.56	45.72	49.59	52.34	58.30
30	14.95	16.79	18.49	40.26	43.77	46.98	50.89	53.67	59.70

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SPECIMEN ASSIGNMENT

954/4

STPM

MATHEMATICS (T) (MATEMATIK (T))

PAPER 4 (KERTAS 4)

MAJLIS PEPERIKSAAN MALAYSIA
(MALAYSIAN EXAMINATIONS COUNCIL)

SIJIL TINGGI PERSEKOLAHAN MALAYSIA
(MALAYSIA HIGHER SCHOOL CERTIFICATE)

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STPM 954/4

Parabolas have many applications in science and technology. A cable hung between two towers of a suspension bridge is usually in the shape of a parabola.

If a parabola is rotated about its axis of symmetry, a parabolic surface is formed which can be used to reflect light. When a ray of light hits the surface of a parabolic concave mirror, the angle between the incident ray and the tangent of the mirror at that point is equal to the angle between the reflected ray and the tangent of the mirror at that point. It is interesting to find out what happens to the reflected light if the incident ray of light is parallel to the principal axis of a parabolic concave mirror. This may be done using precisely drawn graphs of parabolas.

1. (a) Draw the graph of $y^2 = 4x$ for $0 < x < 9$.

(b) Draw as accurately as possible tangents (or normals) for a few suitable values of x and show how the incident rays parallel to the x -axis are reflected. Make a conclusion for this investigation and estimate the coordinates of the focus of the parabola $y^2 = 4x$.

2. (a) Repeat the above procedures for parabolas $y^2 = kx$, where k is a positive constant and $k \neq 4$ and show how to obtain their respective foci.

(b) Deduce, in terms of k , the focus of the parabola $y^2 = kx$, where k is a positive constant.

(c) Discuss the case when k is a negative constant.

3. Investigate and conclude the coordinates of the focus of the parabola $y = kx^2$, where k is a positive constant.

4. A flash light with a parabolic reflecting mirror is positioned such that the axis of symmetry is vertical. The bulb is located at the focus of the mirror and light from this point is reflected outward parallel to the axis of symmetry. The mirror has a diameter of 40 cm and a depth of 20 cm. Where the bulb should be placed relative to the vertex of the mirror?